









Energy dissipation caused by boundary layer instability at vanishing viscosity

Marie Farge, *LMD*, *ENS Paris*,
Natacha Nguyen van yen, Matthias Waidmann and
Rupert Klein, *Institut für Mathematik*, *Freie Universität*, *Berlin*,
Kai Schneider, *I2M*, *Aix-Marseille Université*

GDR Turbulence 2018 Université Nice Sophia Antipolis October 17th 2018



1750: Euler's problem

On 16 May 1748 Euler, president of the Prussian Academy of Sciences, read the problem he proposed for the Prize of Mathematics to be given in 1750:

'Deduce from new principles, as simple as possible, a theory to explain the resistance exerted on a body moving in a fluid, as a function of the body's velocity, shape and mass, and of the fluid's density and compressibility'.

Six mathematicians, including d'Alembert, sent a manuscript, but Euler was not satisfied with them and decided to postpone the prize to 1752.

Grimberg, D'Alembert et les équations aux dérivées partielles en hydrodynamique, Thèse de Doctorat, Université de Paris VII, 1998



Leonhard Euler (1707-1783)

Jean Le Rond d'Alembert (1717-1783)

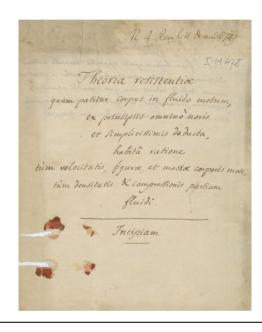


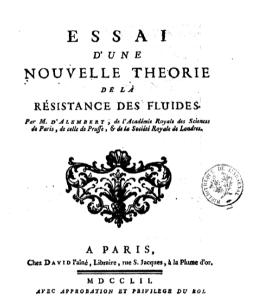


1752: d'Alembert's paradox

D'Alembert was upset and took back his manuscript of 1749, translated it into French and published it in 1752.

1749





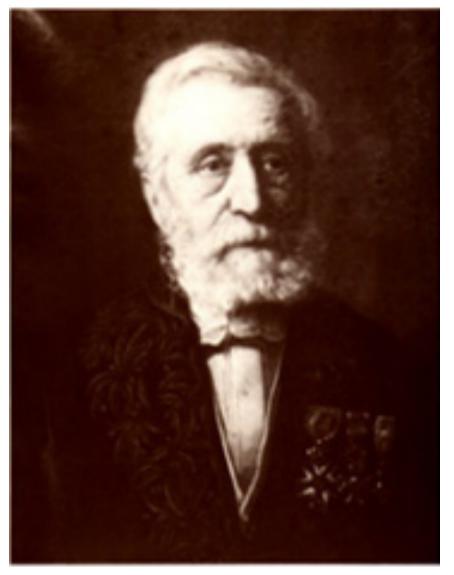
1752

'It seems to me that the theory, developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance, a singular paradox which I leave to future geometers to elucidate.'



Adhémar Jean-Claude Barré de Saint-Venant (1797-1886)

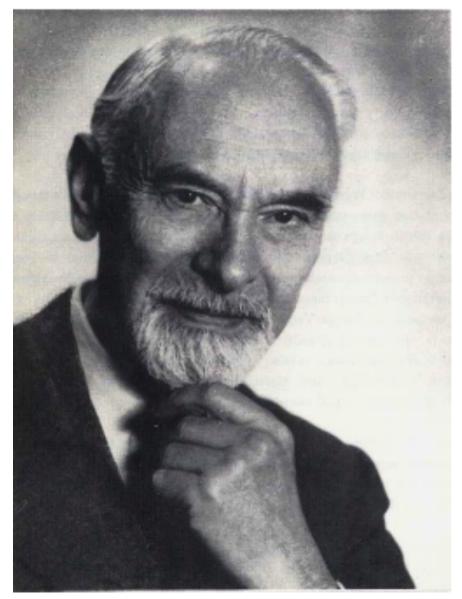
George Stokes (1819-1903)

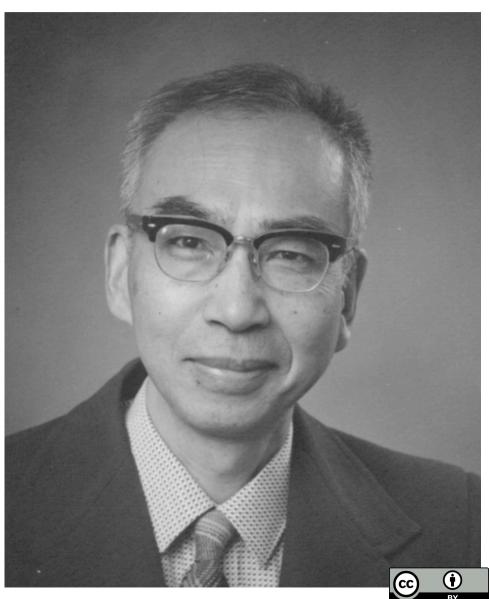




Ludwig Prandtl (1875-1953)

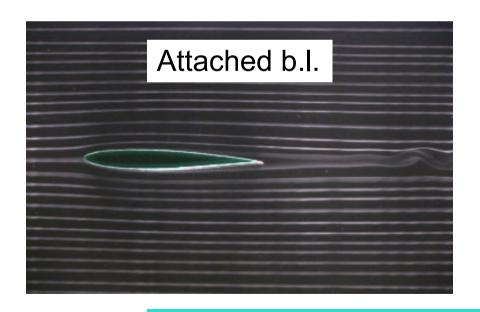
Toshio Kato (1917-1999)

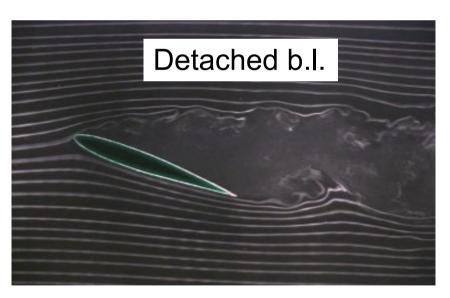




1904: Prandtl's boundary layer theory

- Prandtl (1904) proposed to use Euler equation far from walls and Navier-Stokes equation in boundary layers attached to walls.
- He predicted that the thickness of the boundary layers attached to walls scales as $Re^{-1/2}$,
- Prandtl's theory does not apply if the boundary layers detach.





Prandtl, Über Flüssigkeitsbewegung bei sehr kleiner Reibung, Proceedings of the 3rd ICM in Heidelberg, 484-491, 1904



What is the inviscid limit of Navier-Stokes?

Navier-Stokes equations with no-slip boundary conditions:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla t + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} \\ \nabla \cdot \mathbf{u} = 0 \\ \mathbf{u}_{|\partial\Omega} = \mathbf{0}, \quad \mathbf{u}(0, \cdot) = \mathbf{v} \end{cases} \longrightarrow \mathbf{u}_{\mathrm{Re}}(t, \mathbf{X}) \quad \text{for } v \to 0 \\ \mathrm{Re} \to +\infty \end{cases}$$

$$\mathsf{Re} = \mathsf{VL}v^{-1} \quad \mathsf{Reynolds number}$$

Same initial conditions

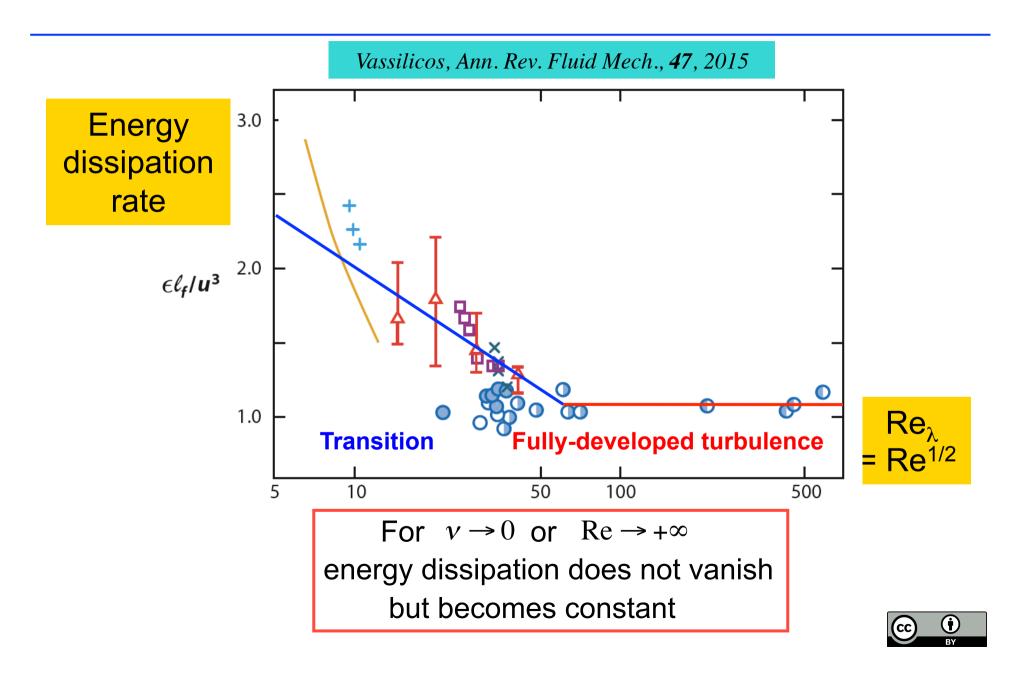


Euler equations with slip boundary conditions:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p \\ \nabla \cdot \mathbf{u} = 0 \\ \mathbf{u}_{|\partial\Omega} \cdot \mathbf{n} = \mathbf{0}, \quad \mathbf{u}(0, \cdot) = \mathbf{v} \end{cases} \longrightarrow \mathbf{u}(t, \mathbf{x}) \quad \begin{cases} \text{for } \\ \mathbf{v} = 0 \\ \text{Re} = +\infty \end{cases}$$



Laboratory experiments



Numerical experiments

Normalized energy dissipation as $v \rightarrow 0$, or Re $\rightarrow \infty$ $\epsilon L/u'^3$ Cao et al Yeung and Zhou Jimenez et al Dissipation Wang et al(decaying) Wang et al(forced) 0 Kaneda et al., 2003 rate present DNS Phys. Fluids, 12, 21-24 Gotoh et al ES DNS 20483 512³ 10243 40963 0.5 (0.41)Strong turbulence Weak 0

Both laboratory and numerical experiments show that the dissipation rate of turbulent flows becomes independent of the fluid viscosity for large Re

500

600

700

800

100

0

200

300

400

 R_{λ}



 $Re_{\lambda} = Re^{1/2}$

1984: Kato's theorem

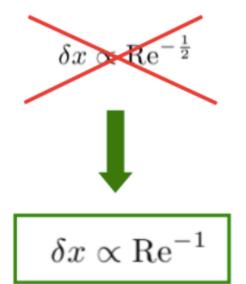
Navier-Stokes solution converges towards the Euler solution, if and only if, energy dissipation vanishes

$$\Delta E_{\text{Re}}(0,T) = \text{Re}^{-1} \int_{0}^{T} dt \int_{\Omega} d\mathbf{x} |\nabla \mathbf{u}(t,\mathbf{x})|^{2} \underset{\nu \to 0}{\longrightarrow} 0,$$

Toshio Kato (1917-1999)

Kato, 1984, Remarks on zero viscosity limit for non stationary Navier-Stokes flows with boundary, MSRI Berkeley

and, if and only if, this happens in a boundary layer whose thickness scales as Re^{-1}

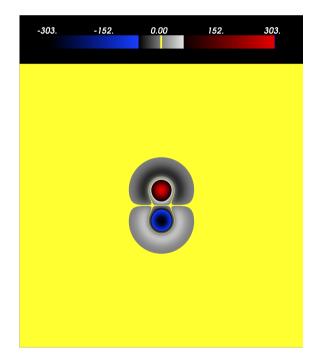


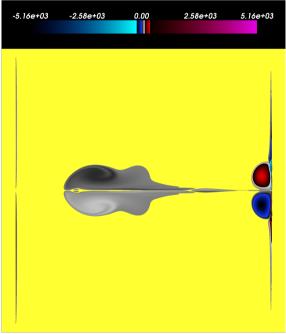
This requires using smaller resolution to compute high Reynolds flows than predicted by Prandtl's theory

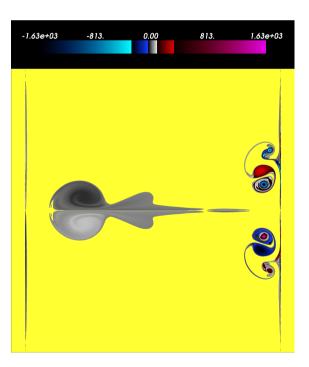


Dipole crashing onto a wall in 2D

DNS Resolution N=8192²







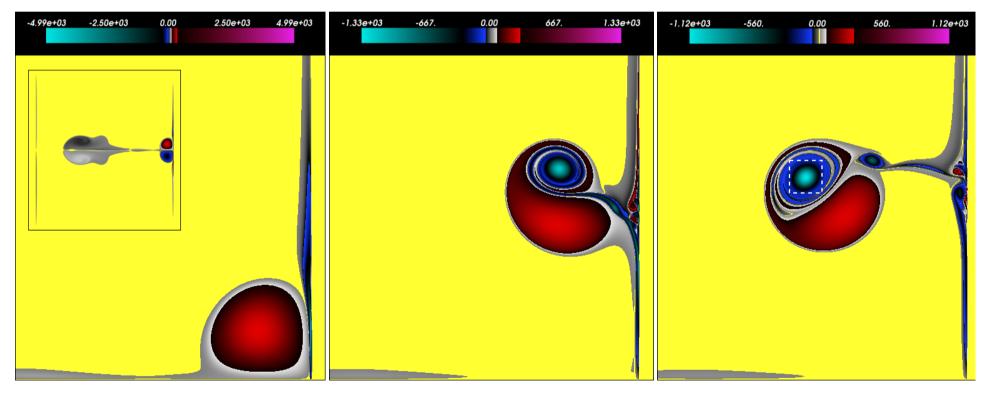


Zoom when boundary layers detach

Resolution N=16384²

Navier-Stokes equations with volume penalization integrated using Fourier

Nguyen van yen, M. F. and Schneider, PRL, **106**(18), 2011



t=0.3

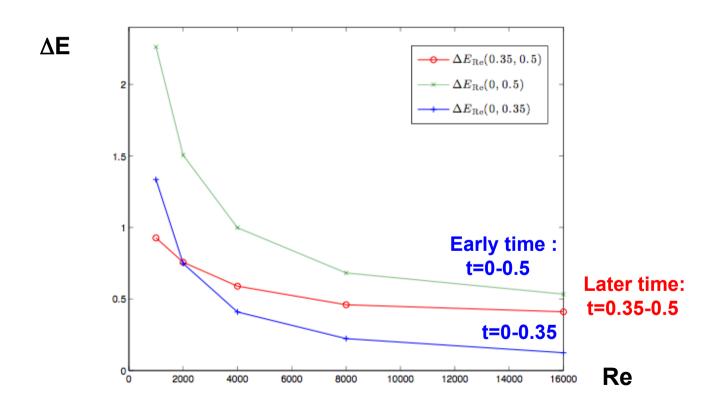
t = 0.4

t=0.5

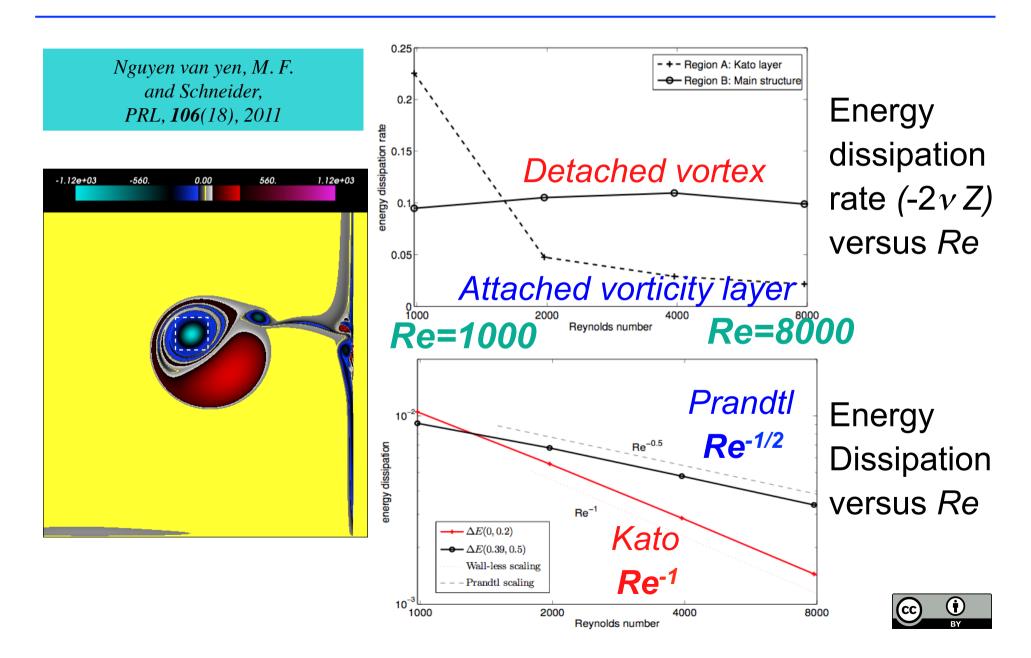


Energy dissipation

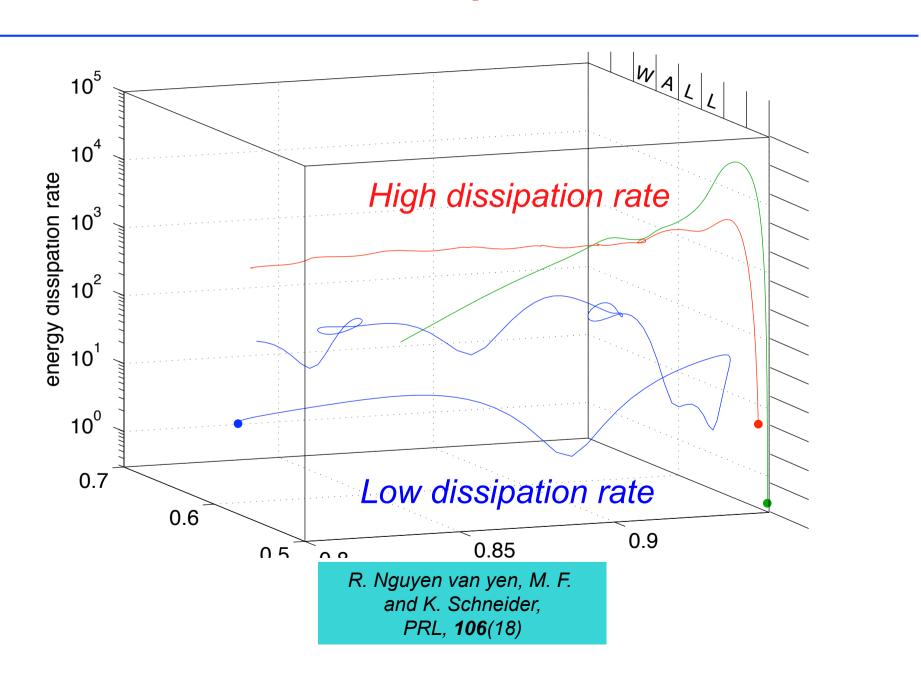
Energy dissipated when the dipole crashes onto the wall at increasing Reynolds numbers



Production of dissipative structures



Production of dissipative structures



Energy Dissipating Structures Produced by Walls in Two-Dimensional Flows at Vanishing Viscosity

Romain Nguyen van yen and Marie Farge LMD-CNRS-IPSL, École Normale Supérieure, Paris, France

Kai Schneider

M2P2-CNRS and CMI, Université d'Aix-Marseille, Marseille, France (Received 13 October 2010; published 3 May 2011)

2013

PHYSICS OF FLUIDS 25, 093104 (2013)

The effect of slip length on vortex rebound from a rigid boundary

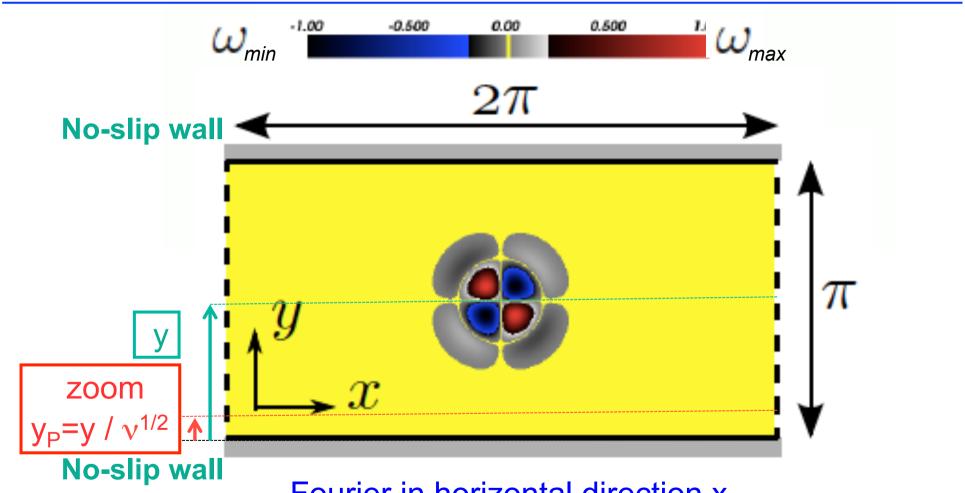
D. Sutherland, 1,a) C. Macaskill, and D. G. Dritschel²

¹School of Mathematics and Statistics, University of Sydney, Sydney 2006, Australia ²School of Mathematics and Statistics, University of St. Andrews, St. Andrews KY16 9SS, United Kingdom

(Received 22 May 2013; accepted 16 August 2013; published online 23 September 2013)



Comparison Navier-Stokes and Euler-Prandtl



Fourier in horizontal direction x

5th order compact finite differences in y

3rd order Runge-Kutta in time



Prandtl equation coupled to Euler

Ansatz for the vorticity field as $\text{Re} \to \infty$:

$$\omega(x,y) = \omega_E(x,y) + \nu^{-1/2}\omega_P(x,\nu^{-1/2}y) + \omega_R(x,y)$$

Prandtl's variable : $y_P = y / v^{1/2}$

$$\partial_t \omega_P + \nabla \cdot (\mathbf{u}_P \omega_P) = \partial_{y_P}^2 \omega_P$$

$$\omega_P(x, y_P, 0) = 0$$

$$\psi_P(x, y_P, t) = \int_0^{y_P} dy_P' \int_0^{y_P'} dy_P'' \omega_P(x, y_P', t)$$

$$\partial_{y_P} \omega_P(x, 0, t) = -\partial_x p_E(x, 0, t),$$

where p_E is the pressure calculated from ω_E which is the vorticity given by Euler equation



Comparison Navier-Stokes and Euler-Prandtl

Navier-Stokes solver

- Fourier in x and compact finite differences of 5th order with non-uniform grid in y.
- Third order Runge-Kutta in t.
- Periodic in x and no-slip boundary conditions in y.

Euler solver

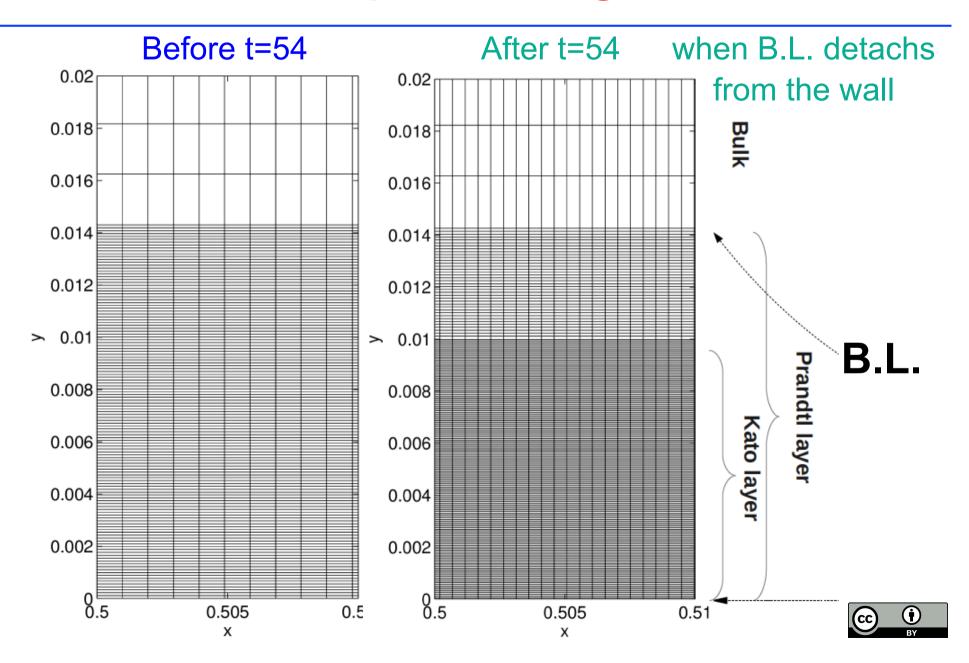
- Fourier with hyperdissipation in x and y.
- Third order Runge-Kutta in t.
- Mirror-symmetry around y=0 to impose boundary conditions.

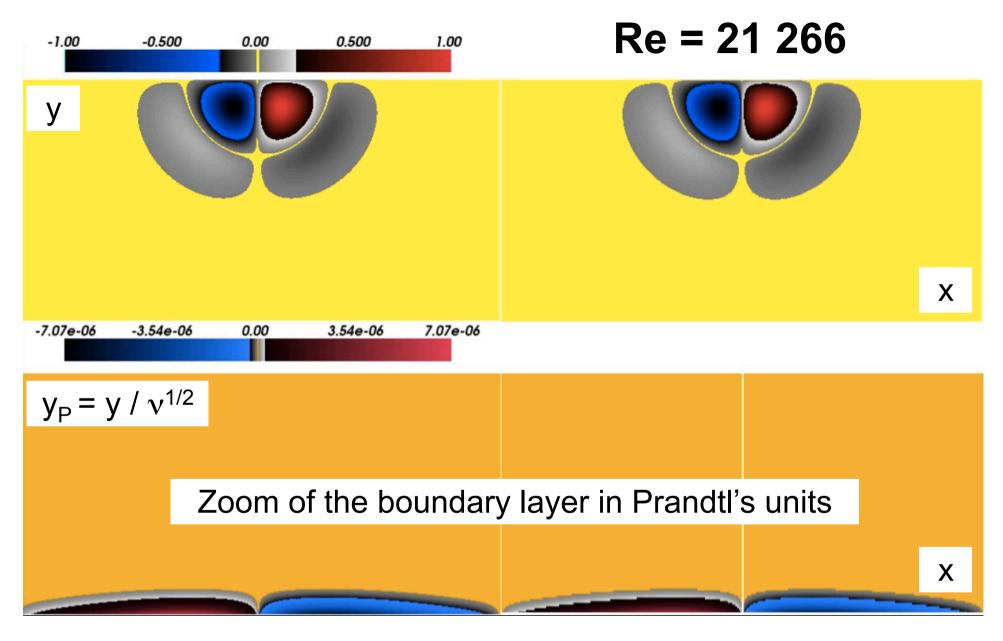
Prandtl solver

- Second order finite differences in x and y.
- Second order semi-implicit Runge-Kutta in t.
- Neumann boundary condition at y=0 when inverting.



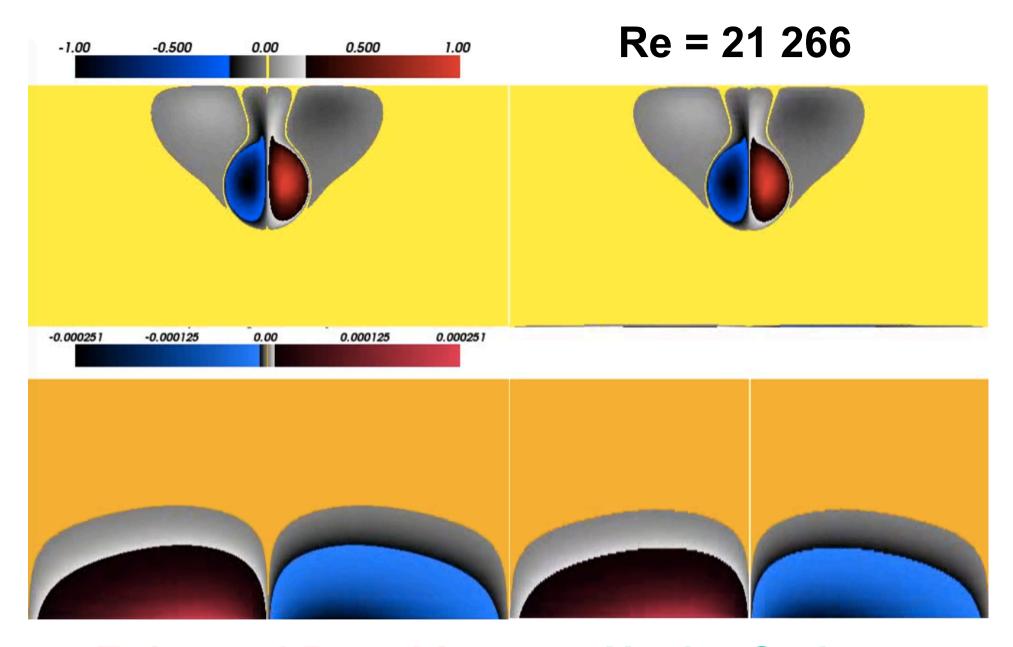
Computational grid





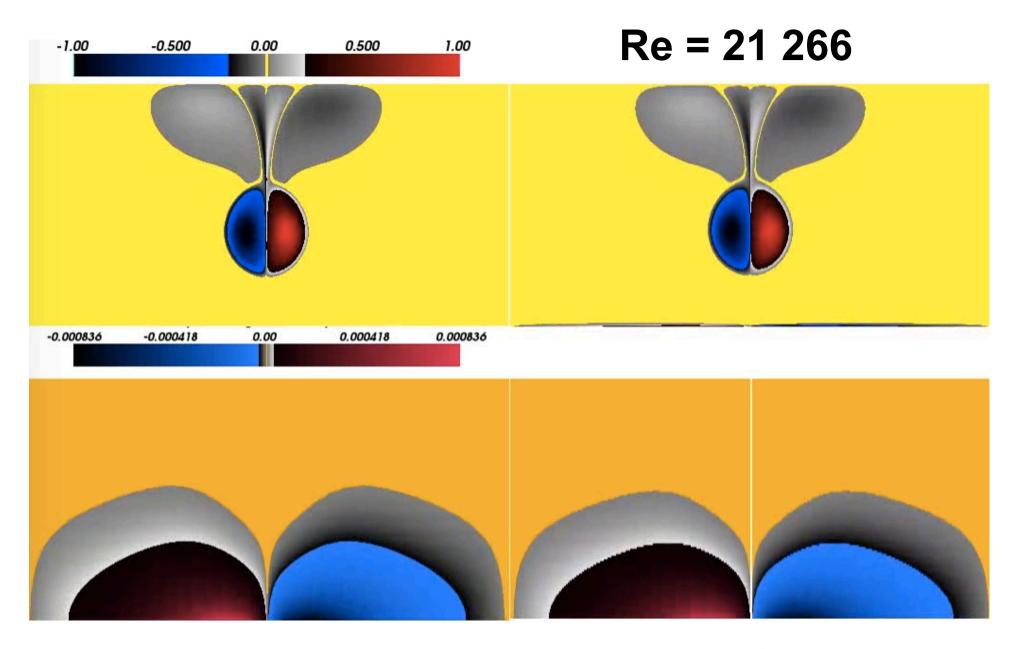
Euler and Prandtl





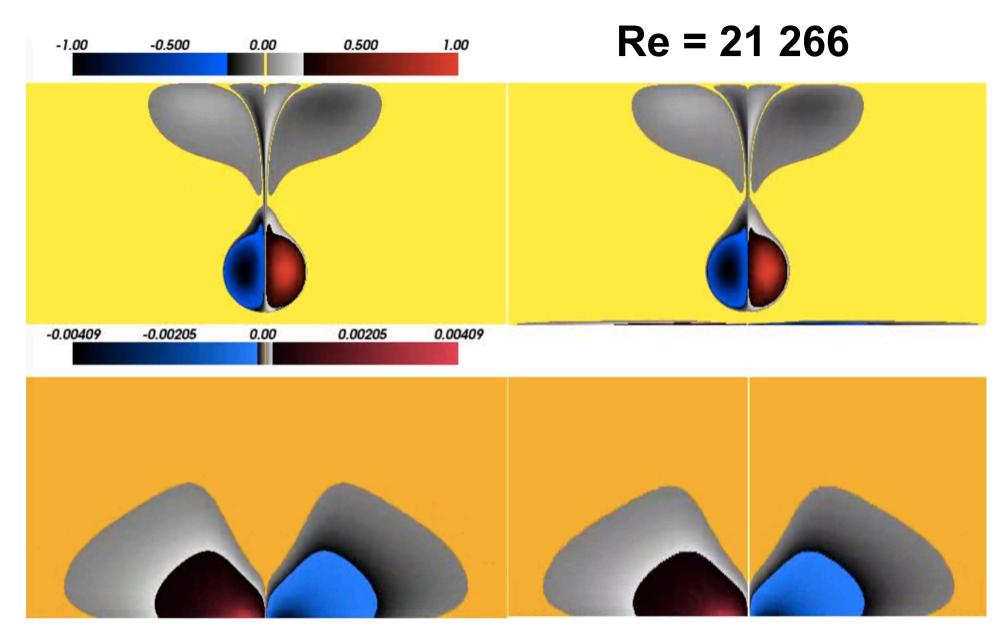
Euler and Prandtl





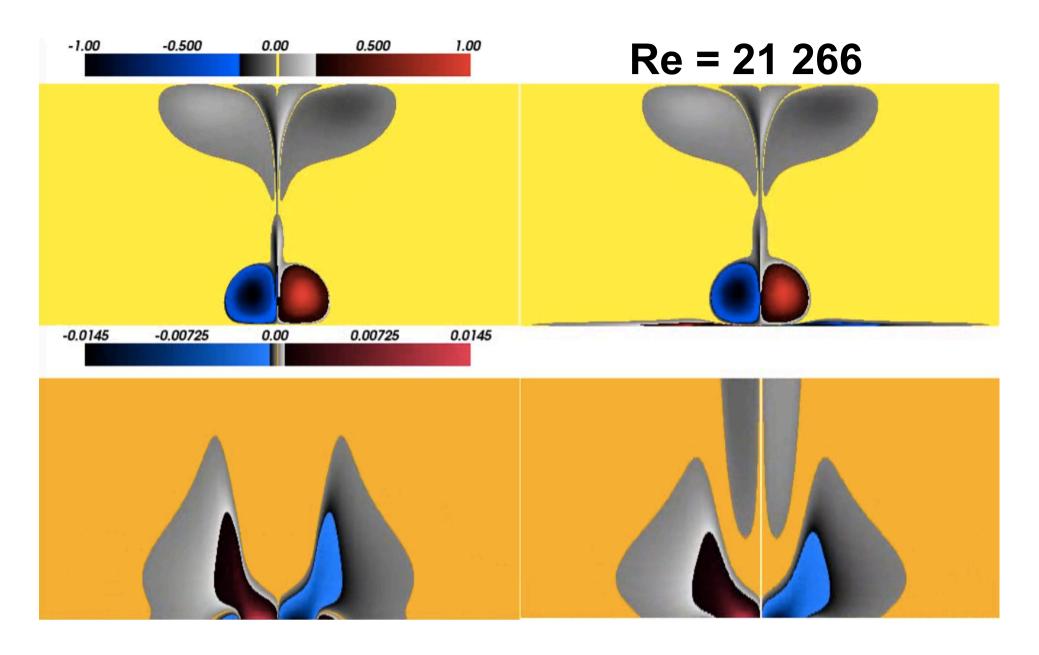
Euler and Prandtl





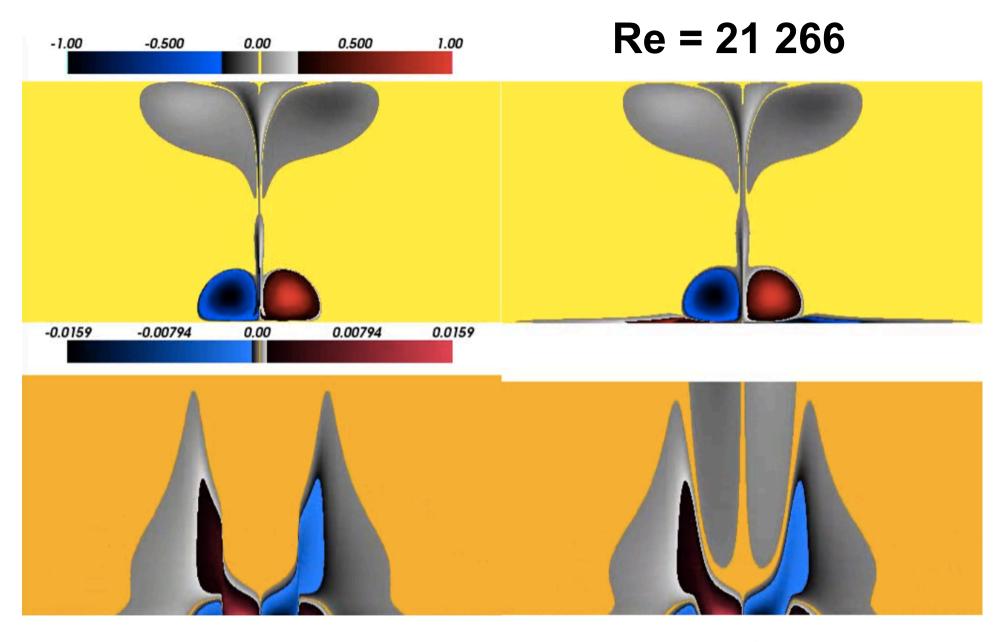
Euler and Prandtl





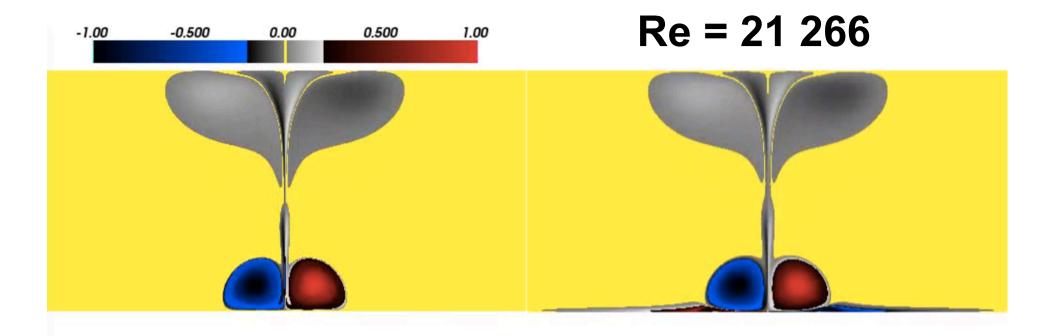
Euler and Prandtl





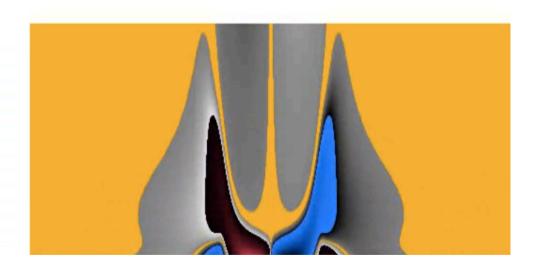
Euler and Prandtl





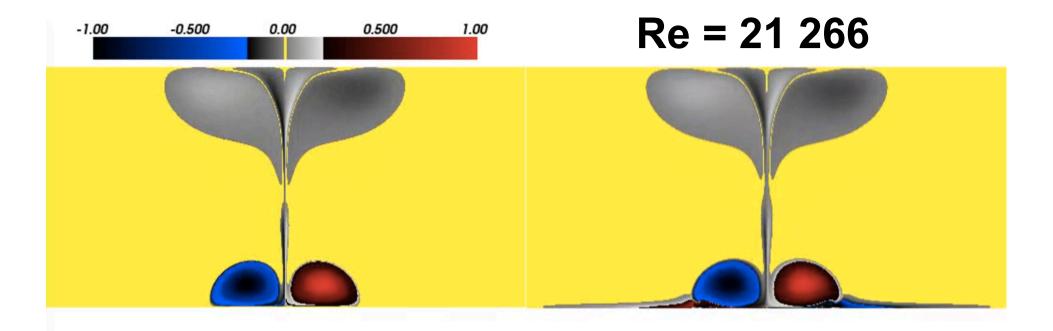
Prandtl's solution becomes singular at t= 55.8

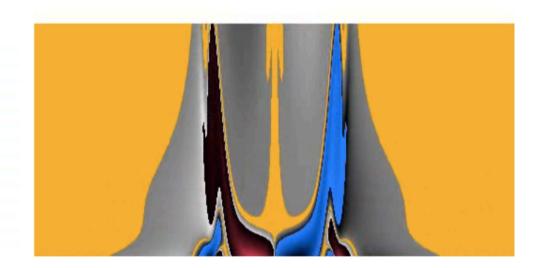
> L. L. van Dommelen and S. F. Shen., 1980 J. Comp. Phys., **38**(2)



Euler

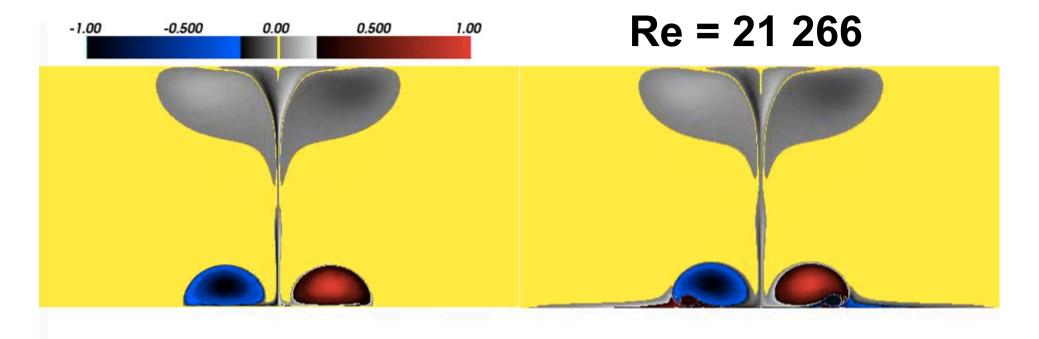
Navier-Stokes

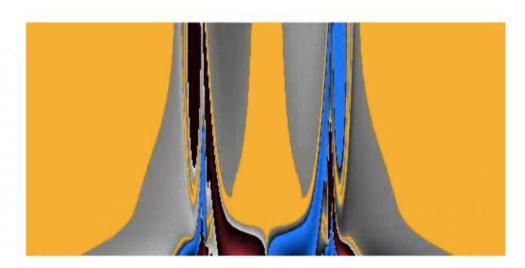




Euler

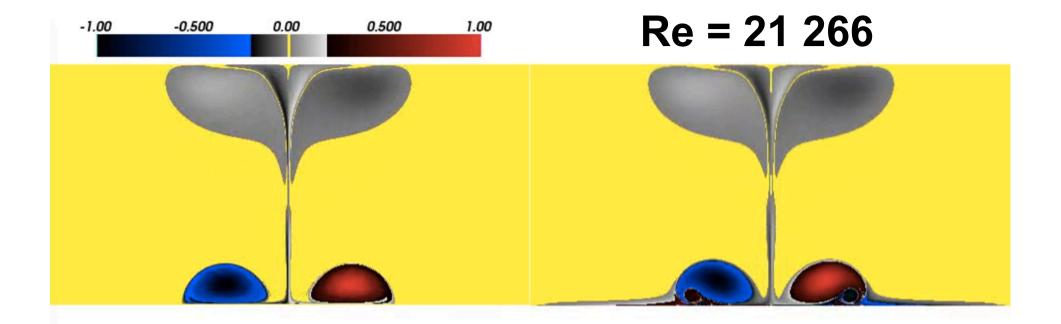


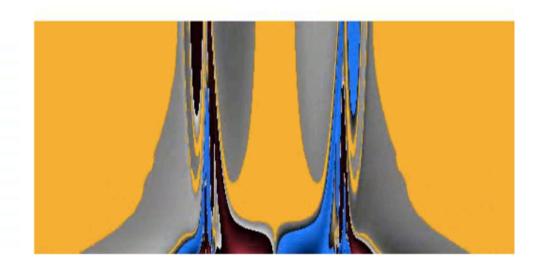




Euler

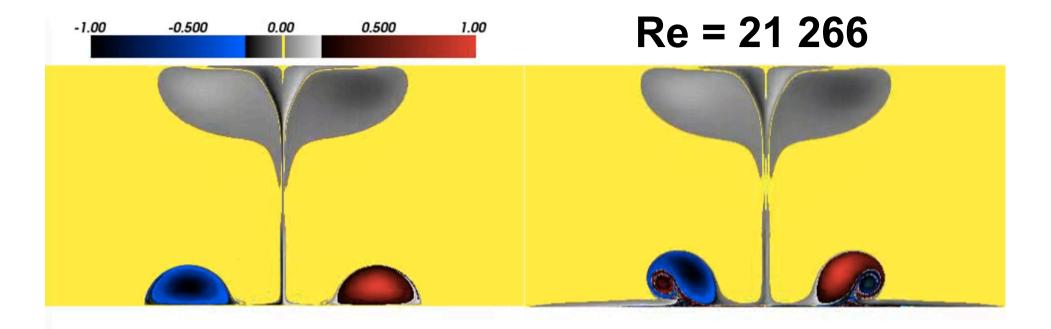


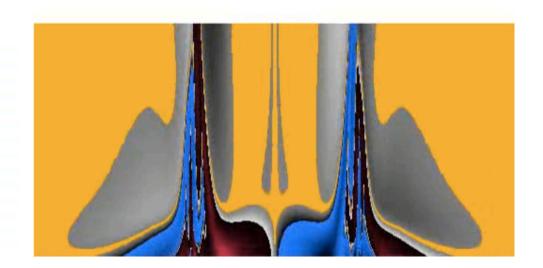




Euler

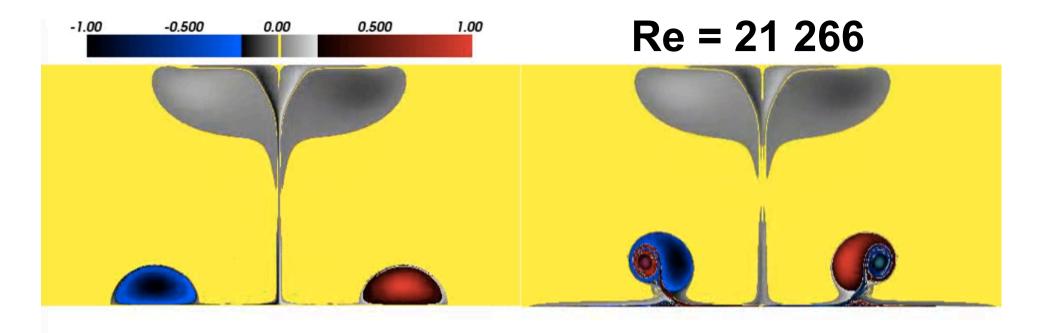


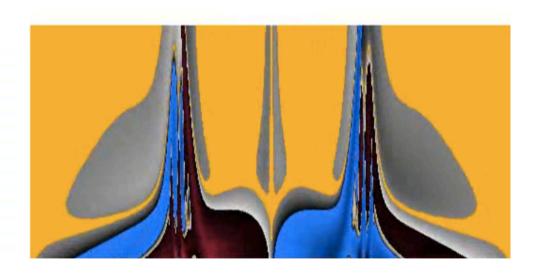




Euler

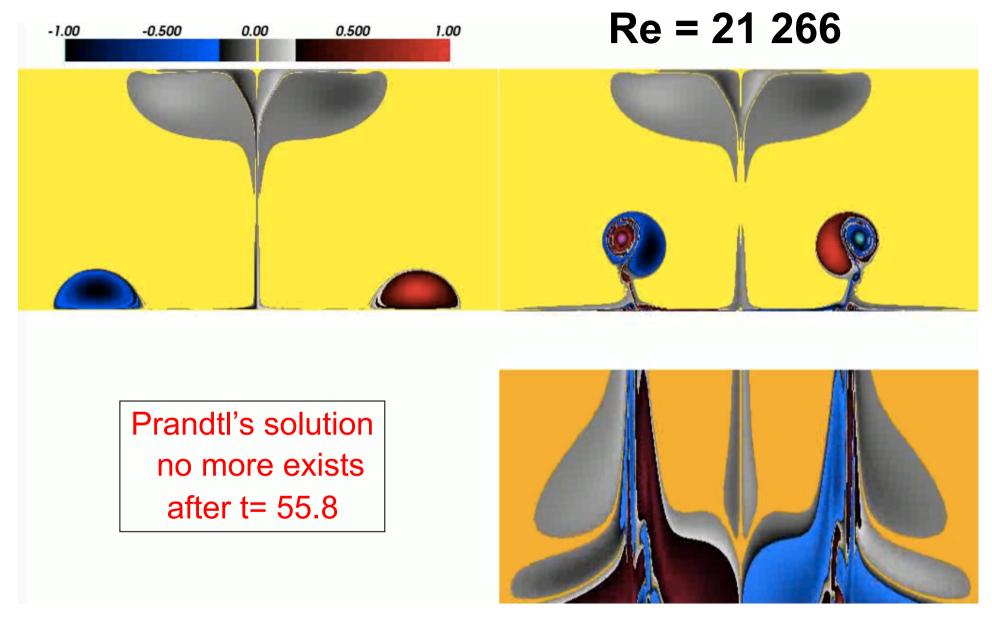






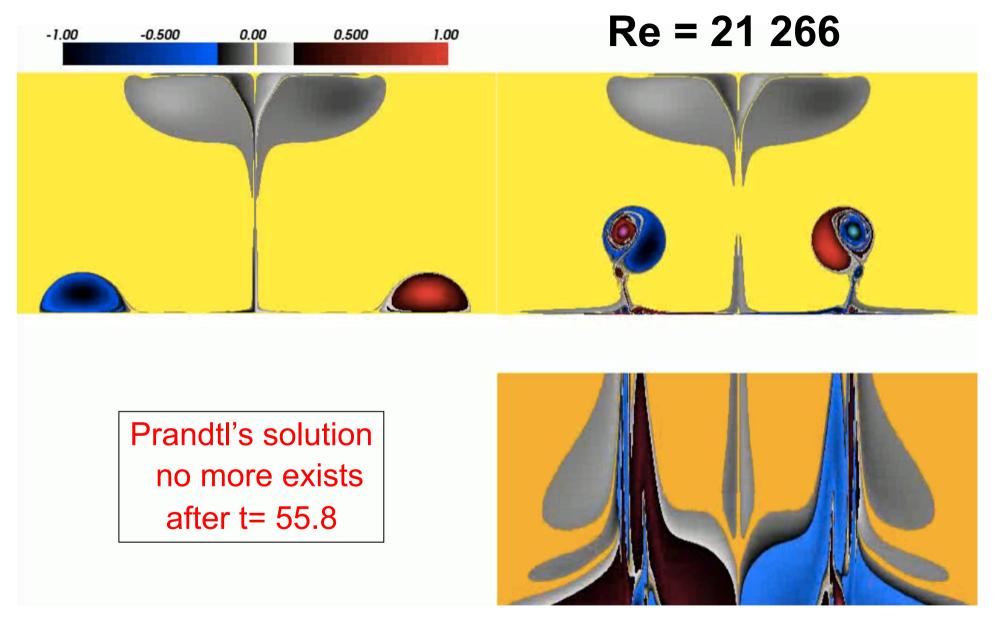
Euler





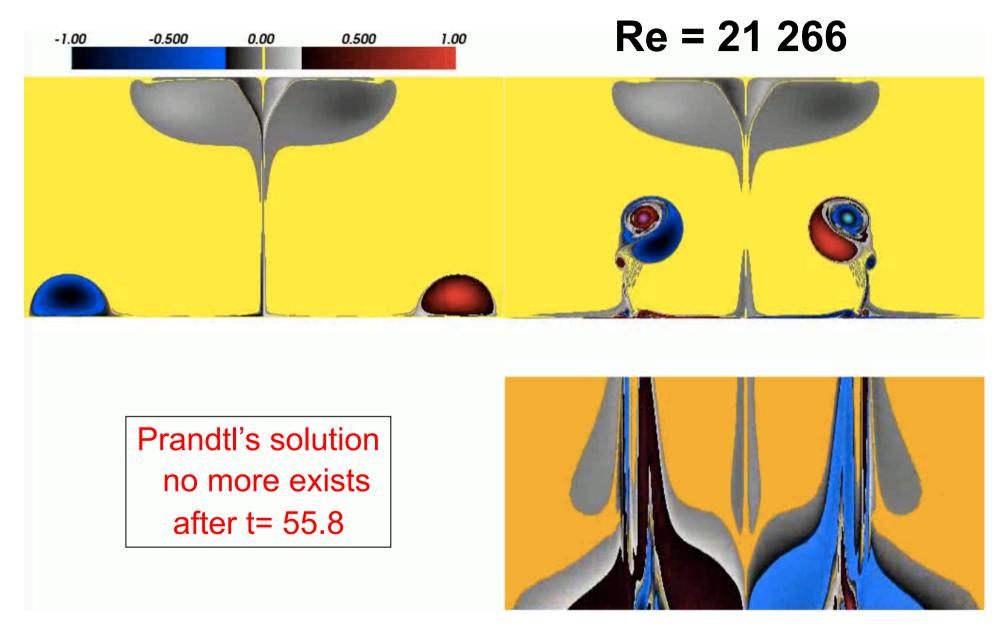
Euler





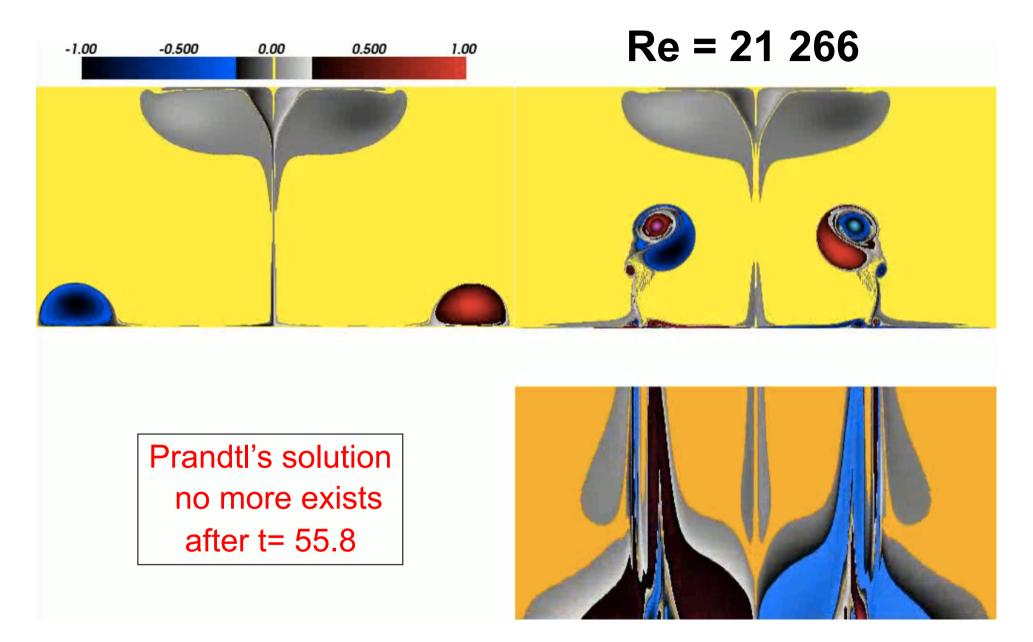
Euler





Euler





Euler





Prandtl's singularity

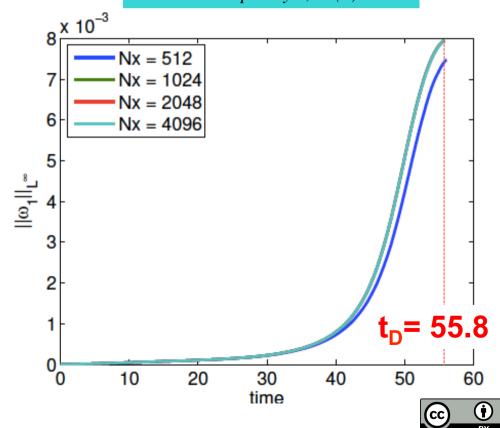
Prandtl equation has well-known finite time singularity

- $|\partial_x \omega_1|$ and $u_{1,y}$ blows up,
- ω_1 remains bounded.

This is observed in our computations as expected,

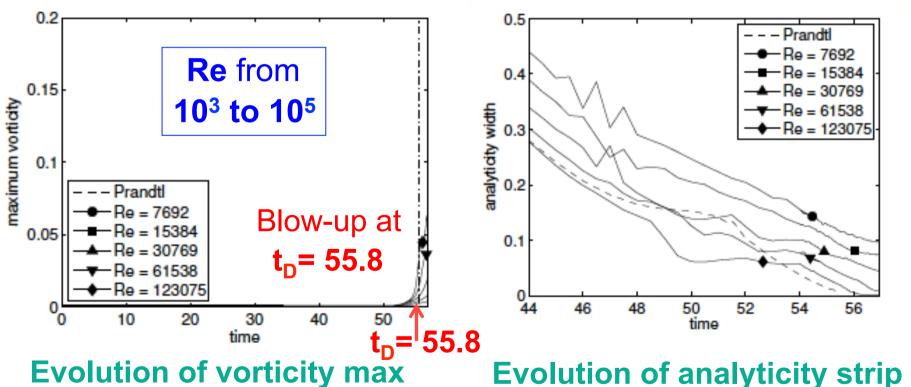
for
$$t o t_D \simeq 55.8$$

L. L. van Dommelen and S. F. Shen., 1980 J. Comp. Phys., **38**(2)



Prandtl solution's blow-up at t_D=55.8

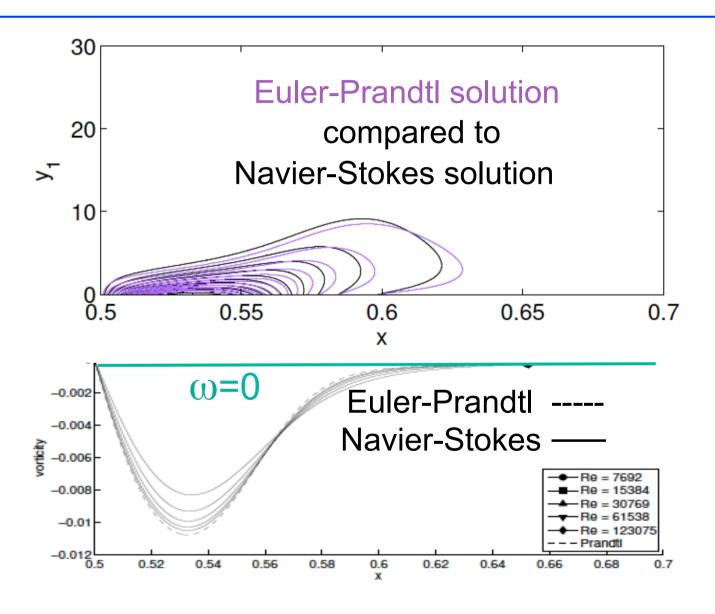
According to Kato's theorem, and since ω_1 remains bounded uniformly until t_D , we expect that $\mathbf{u}_{\nu} \xrightarrow[\nu \to 0]{L^2} \mathbf{u}_0$ uniformly on $[0, t_D]$.



Show convergence!

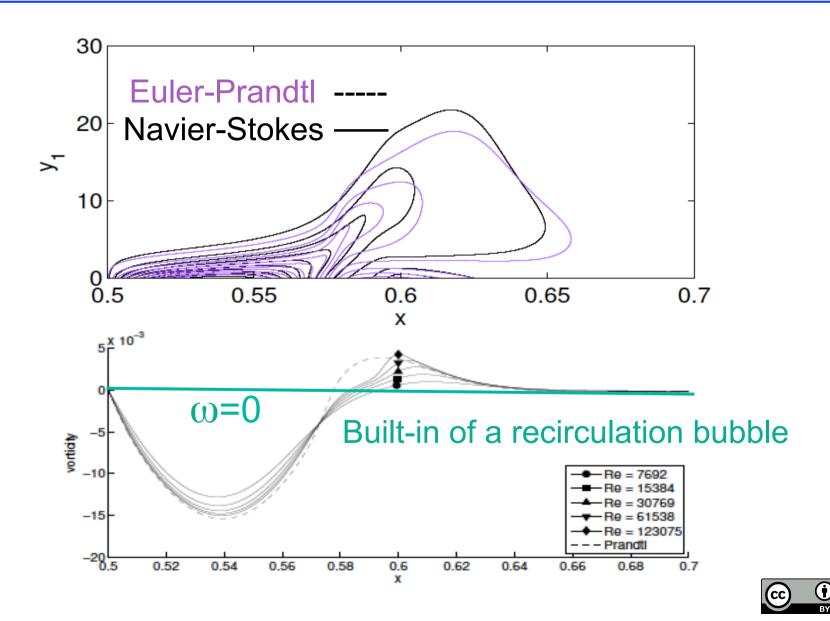


Vorticity along the wall at t=50 < t_D

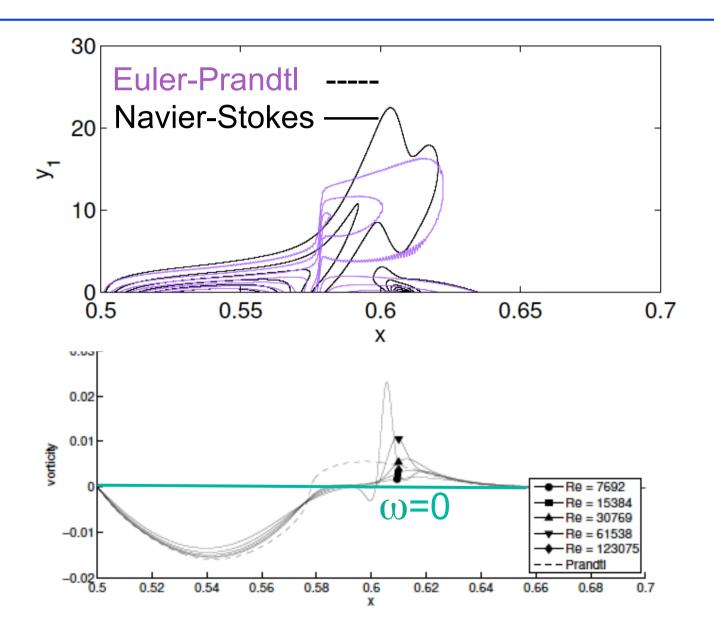




Vorticity along the wall at t=54 < t_D

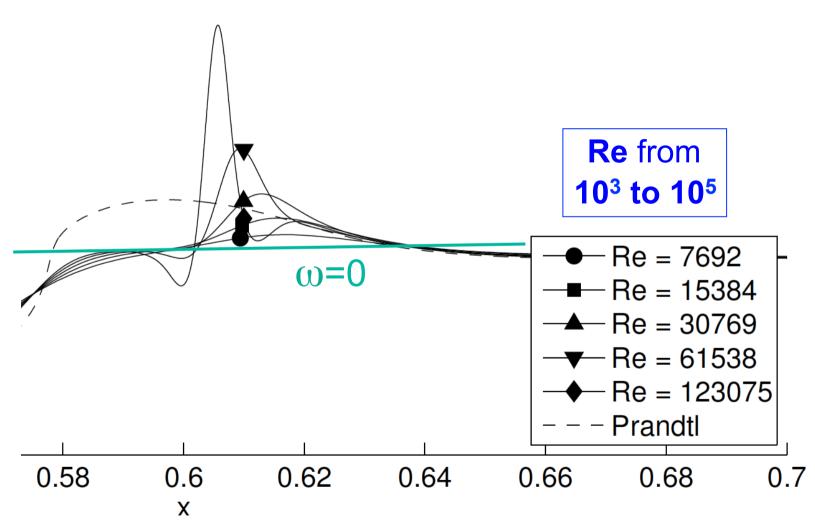


Vorticity along the wall at t=55 < t_D



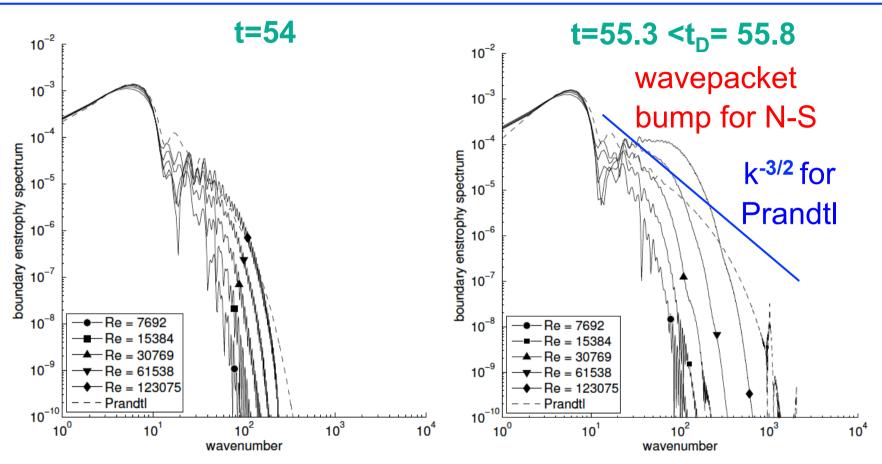


Vorticity along the wall at t=55.3 < t_D





Spectrum of the boundary vorticity

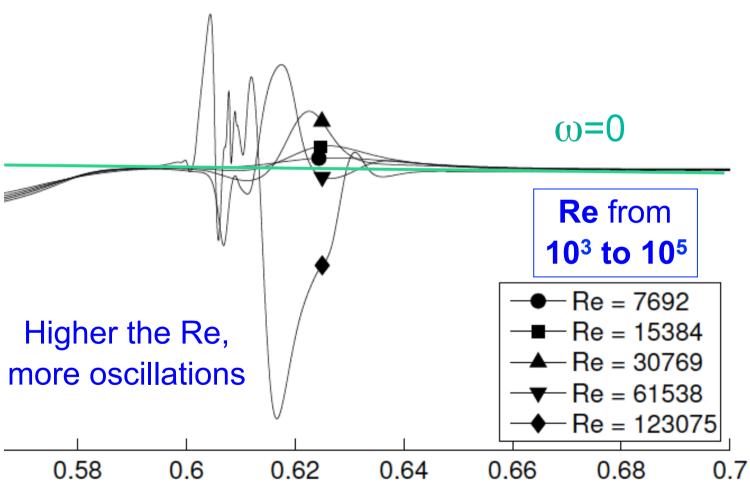


The Prandtl's solution behaves as k^{-3/2} for large k, consistent with the build-up of a jump singularity of vorticity along the wall, while Navier-Stokes develops a bump which spreads in k with Re.



Vorticity along the wall at $t=57.5 > t_D$

Production of a wavepacket of vorticity when and where boundary layer detaches

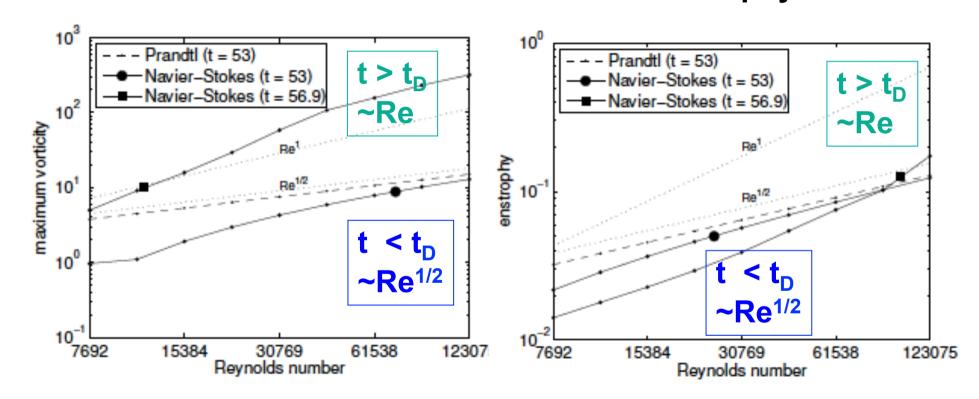




Scaling from Re=7692 to 123075

Vorticity max

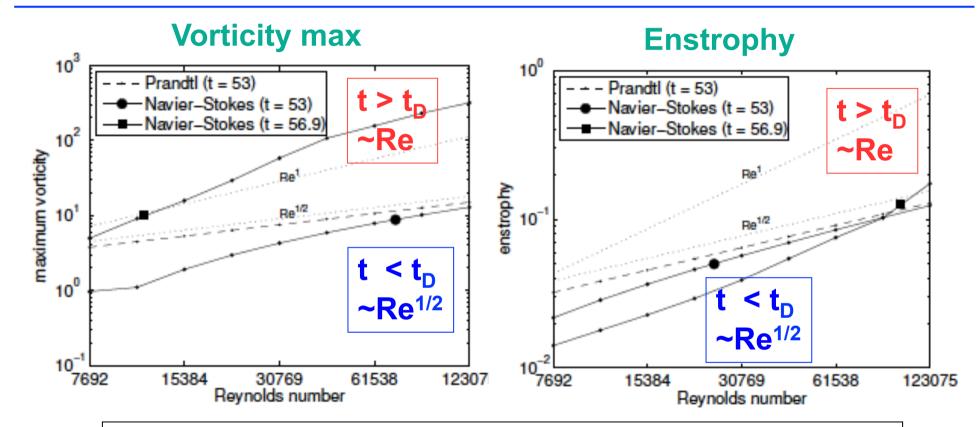
Enstrophy



We observe Prandtl's scaling in Re^{1/2} before t_D~ 55.8 and Kato's scaling in Re after.



Scaling from Re = 7692 to 123075



We observe Prandtl's scaling in Re $^{1/2}$ before $t_D \sim 55.8$ and Kato's scaling in Re after t_D .

Nguyen van yen, M. F. and Schneider, Phys. Rev. Lett., **106**(18), 2011 Nguyen van yen, Waidmann, Klein, M. F. and Schneider, J. Fluid Mech., 849, 676-717, 2018



Relation to the von Karman wall law

In turbulent boundary layers the mean velocity profile satisfies

$$\langle U(y) \rangle \simeq \frac{U_{\tau}}{K_{\text{karman}}} \log \left(\frac{yU_{\tau}}{\nu} \right)$$

the so called 'log law', where

$$U_{\tau} = \sqrt{\nu \left\langle \left. \frac{\mathrm{d}U}{\mathrm{d}y} \right|_{y=0} \right\rangle}$$

is the friction velocity.

This shows that the bulk velocity and U_{τ} have the same scaling with Re. This can be seen as a statistical signature of a boundary layer thickness Re⁻¹, which is consistent in some sense with the existence of a Kato layer.

T. von Karman, Uber laminare und turbulente Reibung. Z. ang. Math. Mech. 1 (4), 233{252, 1921



Attached / detached boundary layer

- The Prandtl solution becomes singular at t_Dwhen BL detaches.
- The Navier-Stokes solution converges uniformly to the Euler solution before the boundary layer remains attached and ceases to converge after the boundary layer detaches.
- The detached BL has spatial scales as fine as Re⁻¹, whic are produced in different directions and not only parallel to the wall, while the attached BL is parallel to the wall and scales as Re^{-1/2}.
- The maximal vorticity of Navier-Stokes solution does not appear at the same location where Prandtl's solution becomes singular.
 It apprears not on the wall but near le wall.

Nguyen van yen, Waidmann, Klein, M. F. and Schneider, J. Fluid Mech., 849, 676-717, 2018



Interpretation of the von Karman wall law

- The velocity gradient du/dy at the wall scales like Re, which can be seen as the statistical signature of the existence of a boundary layer of thickness Re in the neighborhood of the wall.
- Hence, the log-law, which is obtained from experimental results, is consistent with the existence of a Kato layer. This connection can be made in a phenomenological way without invoking the Kolmogorov scale and cascade.
- Our results may help in investigating rigorous foundations to the phenomenological wall law of von Karman.

Nguyen van yen, Waidmann, Klein, M. F. and Schneider, J. Fluid Mech., 849, 676-717, 2018



Open questions

Numerical results suggest that a new asymptotic description of the flow beyond the breakdown of the Prandtl regime is possible. Studying it might help to answer the following questions:

- Would Navier-Stokes solution looses smoothness after t_D?
- -Would it converges to a weak singular dissipative solution of Euler's equation analog to dissipative shocks in Burgers solution?
- How can such a weak solution be approximated numerically?

This might lead to a new resolution of d'Alembert's paradox in terms of the production of weak singular dissipative structures due to the interaction of fully-developed turbulent flows with walls.

J. Leray, 1934 Sur le mouvement d'un fluide visqueux, Acta Mathematica, **63** C. de Lellis and L. Székzlyhidi, 2010 Archives Rational Mechanics and Analysis, 195(1), 221-260



Our article is published in Open Access

J. Fluid Mech. (2018), vol. 849, pp. 676–717. © Cambridge University Press 2018 676
This is an Open Access article, distributed under the terms of the Creative Commons Attribution licence (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted re-use, distribution, and reproduction in any medium, provided the original work is properly cited. doi:10.1017/jfm.2018.396

Energy dissipation caused by boundary layer instability at vanishing viscosity

Natacha Nguyen van yen¹, Matthias Waidmann¹, Rupert Klein¹, Marie Farge²,† and Kai Schneider³

¹Institut für Mathematik, Freie Universität Berlin, Arnimallee 6, 14195 Berlin, Germany
²LMD-CNRS, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris CEDEX 5, France
³Institut de Mathématiques de Marseille, Aix-Marseille Université and CNRS, Marseille, France

(Received 12 July 2017; revised 4 March 2018; accepted 16 April 2018)



CUP asked us 2200 € and our copyright!

Journal - Open Access form

Please complete both Sections A and B, sign, and return a scanned copy of this form via email to journalscopyright@cambridge.org as soon as possible. By completing, signing and returning this form you hereby agree to the Terms and Conditions attached (Form.OA.14.1).

Journal of Fluid Mechanics

n consider	ration of the publication in Journal of Fluid Mechanics of the contribution entitled:
	by (all authors' names):
Section A	- Assignment of Copyright (fill in either part a.1 or a.2 or a.3)

a.1 To be filled in if copyright belongs to you

I/we hereby assign to Cambridge University Press, full copyright in all forms and media in the said contribution, including in any supplementary materials that I/we may author in support of the online version.

Section B - Warranty and disclosure of conflict of interest (fill in both sections b.1 and b.2)

b.1 Warranty

I/we warrant that I am/we are the sole owner or co-owners of the contribution and have full power to make this agreement, and that the contribution has not been previously published, contains nothing that is in any way an infringement of any existing copyright or licence, or duty of confidentiality, or duty to respect privacy, or any other right of any person or party whatsoever and contains nothing libellous or unlawful; and that all statements purporting to be facts are true and that any recipe, formula, instruction or equivalent published in the Journal will not, if followed accurately, cause any injury or damage to the user. I/we further warrant that permission for all appropriate uses has been obtained from the copyright holder for any material not in my/our copyright including any audio and video material, that the appropriate acknowledgement has been made to the original source, and that in the case of audio or video material appropriate releases have been obtained from persons whose voices or likenesses are represented therein. I/we attach copies of all permission and release correspondence. I indemnify and keep Cambridge University Press, indemnified against any loss, injury or damage (including any legal costs and disbursements paid by them to compromise or settle any claim) occasioned to them in consequence of any breach of these warranties.

..



We refused to give our full copyright

Journal of Fluid Mechanics In consideration of the publication in Journal of Fluid Mechanics of the contribution entitled: Lucy Line to Lucy Line t

I/we hereby assert my/our moral rights in accordance with the UK Cepyright Designs and Patents Act (1988).

Signed (tick one)

the sole author(s)

one author authorised to execute this transfer on behalf of all the authors of the above article unless any authors are Government employees (see section a.3 below)

Name (block letters)

Institution/Company

Signature:

Date:

D

Section B – Warranty and disclosure of conflict of interest (fill in both sections b.1 and b.2)

b.1 Warranty

I/we warrant that I am/we are the sole owner or co-owners of the contribution and have full power to make this agreement, and that the contribution has not been previously published, contains nothing that is in any way an infringement of any existing copyright or licence, or duty of confidentiality, or duty to respect privacy, or any other right of any person or party whatsoever and contains nothing libellous or unlawful; and that all statements purporting to be facts are true and that any recipe, formula, instruction or equivalent published in the Journal will not, if followed accurately, cause any injury or damage to the user. I/we further warrant that permission for all appropriate uses has been obtained from the copyright holder for any material not in my/our copyright including any audio and video material, that the appropriate acknowledgement has been made to the original source, and that in the case of audio or video material appropriate releases have been obtained from persons whose voices or likenesses are represented therein. I/we attach copies of all permission and release correspondence. I indemnify and keep Cambridge University Press, indemnified against any loss, injury or damage (including any legal costs and disbursements paid by them to compromise or settle any claim) occasioned to them in consequence of any breach of the

We asked CUP to recover our copyright

CUP proposed to return our copyright and publish an erratum in the next JFM issue

ERRATUM

Energy dissipation caused by boundary layer instability at vanishing viscosity – ERRATUM

Natacha Nguyen van yen, Matthias <u>Waidmann</u>, Rupert Klein, Marie Farge and Kai Schneider

doi.org/10.1017/jfm.2018.396, Published by Cambridge University Press 26 June 2018

In the original version of this article (Nguyen van Yen et al., 2018), the copyright was incorrectly transferred to Cambridge University Press. The authors retains the copyright of this work. The copyright line has been updated in the original to the following:

© The Authors, 2018



http://dissem.in to foster Open Access

Welcome to dissemin

Dissemin detects papers behind pay-walls and invites their authors to upload them in one click to an open repository.

Type here the first name and family name of a researcher from any discipline

Search

Green open access

Many researchers do not use their right to make their papers freely available online, in addition to the paywalled version offered by traditional publishers.

This forces libraries to buy overpriced electronic subscriptions to journals, when they can afford them at all.



Open repositories

Uploading your papers on your own webpage is not enough. Such copies are less stable and harder to find than documents uploaded to well-indexed repositories.

Dissemin searches for copies of your papers in a large collection of open repositories and tells you which ones cannot be accessed.

Dissem.in crawls the metadata of about 100 Millions research articles

FAQ API Terms of Service

Unavailable

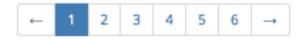
Who are we? Donate Partners hello@dissem.in @disseminOA GitHub

Change language Q

English



Anyone can download for free any article which is already in Open Access, wherever it is stored:





Seung-Bu Park, Pierre Gentine, Kai Schneider, Marie Farge

2016

Coherent Structures in the Boundary and Cloud Layers: Role of Updrafts, Subsiding Shells, and Environmental Subsidence



American Meteorological Society, Journal of the Atmospheric Sciences, 2016.



Frank G. Jacobitz, Kai Schneider, Wouter J. T. Bos, Marie Farge

Structure of sheared and rotating turbulence: Multiscale statistics of Lagrangian and Eulerian accelerations and passive scalar dynamics

Download | American Physical Society, Physical Review E, 1(93), 2016.



Marie Farge, Kai Schneider

2015

Wavelet transforms and their applications to MHD and plasma turbulence: a review



Cambridge University Press (CUP), Journal of Plasma Physics, 06(81), 2015.

Researcher

Marie Farge

- @ 0000-0002-4445-8625
- * École normale supérieure
- ☆ Département de géosciences
- 106 publications



- Available from the publisher 24
- Available from the author 55
- Could be shared by the authors 16
 - Unknown/unclear sharing policy 9
- Publisher forbids sharing 2

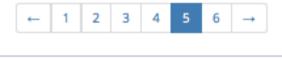
Refine search

By document type:

- Journal article
- Proceedings article
- Book chapter
- Book
- Journal issue



Any author can upload for free his/her articles which are not yet in Open Access:



Marie Farge, Kai Schneider, Giulio Pellegrino, Alan A. Wray, Robert S. Rogallo

Coherent vortex extraction in three-dimensional homogeneous turbulence: Comparison between CVS-wavelet and POD-Fourier decompositions



◆ Upload | American Institute of Physics, Physics of Fluids, 10(15), 2003.



Kai Schneider, Marie Farge

Coherent Vortex Simulation (CVS) of 2D bluff body flows using an adaptive wavelet method with penalisation



▲ Upload | Springer Verlag, Notes on Numerical Fluid Mechanics and Multidisciplinary Design,



Bartosz Protas, Kai Schneider, Marie Farge

Geometrical alignment properties in Fourier- and waveletfiltered statistically stationary two-dimensional turbulence



Physical Review E, 4(66), 2002.



Kai Schneider, Marie Farge

Adaptive Wavelet Simulation of a Flow around an Impulsively Started Cylinder Using Penalisation

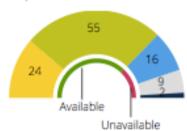
Researcher

Marie Farge

- © 0000-0002-4445-8625
- ★ École normale supérieure
- ☆ Département de géosciences
- 106 publications

2003

2002



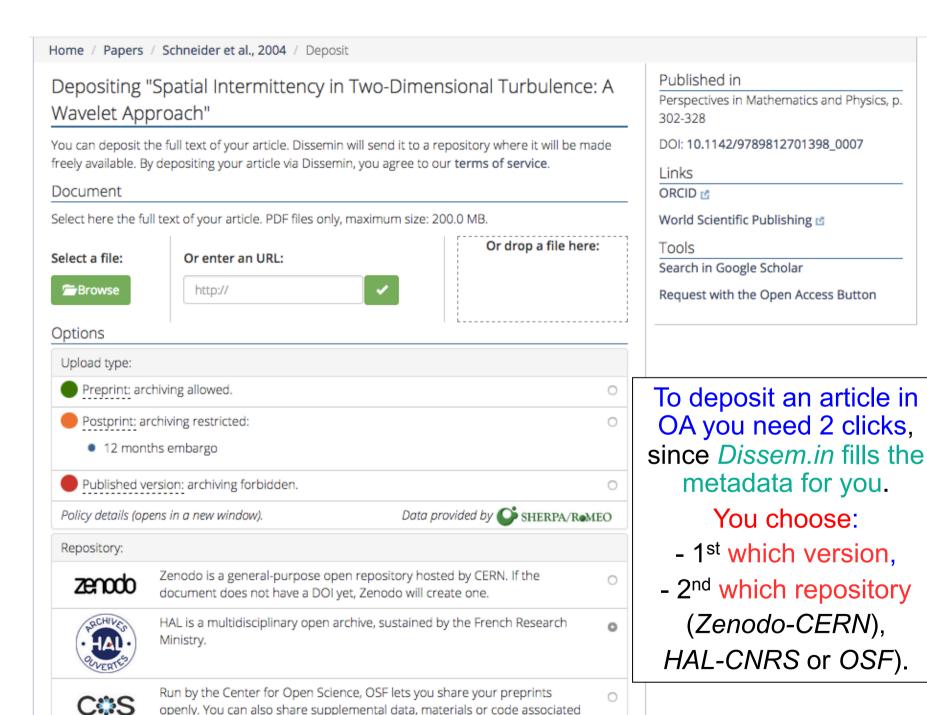
- Available from the publisher 24
- Available from the author 55
- Could be shared by the authors 16
- Unknown/unclear sharing policy 9
- Publisher forbids sharing 2

Refine search

By document type:

- lournal article
- Proceedings article
- Book chapter
- Book
- lournal issue
- Proceedings
- Entry
- Poster
- Report
- Thesis
- Dataset
- Preprint
- Other document



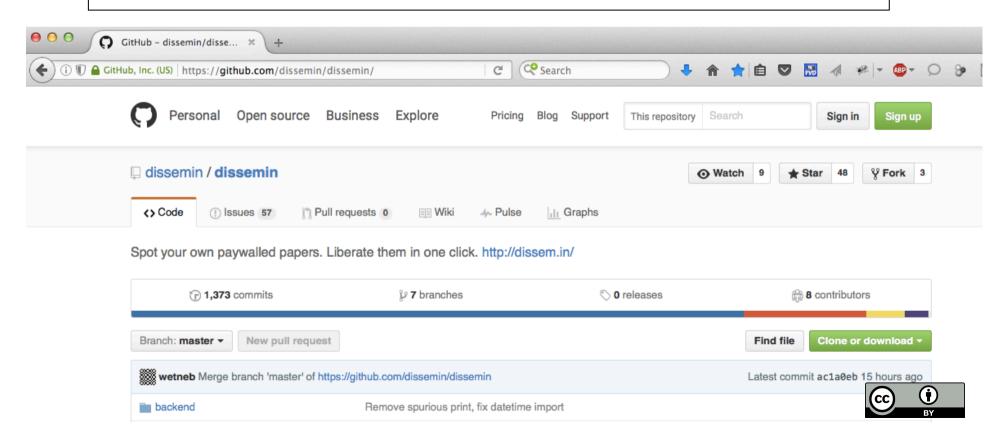


with your article.



The source of *Dissem.in* is free on *GitHub*

Dissem.in is written in Python and published under the viral licence AFFERO GPL 3 which allows everyone to use, modify and distribute the source code, under the condition that the source of the new version and the source of the website to access it should have the same licence.



The team CAPSH develops Dissem.in

Dissem.in runs on three servers which are rented for 100 €/month.

It is financed by the non-profit association Committee for the Accessibility of Publications in Sciences and Humanities (CAPSH), created in 2015 by four students and a researcher from ENS Paris):



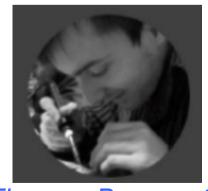
Antonin Delpeuch,
the main developer of Dissem.in,
who was nominated in 2016
Europe's Open Access Champion by











Pablo Rauzy Marie Fai

Thomas Bourgeat

Centre Mersenne to publish in Open Access



https://www.centre-mersenne.org



<marie.farge@ens.fr> http://wavelets.ens.fr http://openscience.ens.fr/MARIE_FARGE



http://dissem.in
http://association.dissem.in
https://github.com/dissemin

https://www.centre-mersenne.org

