

Energy dissipation caused by boundary layer instability at vanishing viscosity

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1750: Euler's problem

On 16 May 1748 Euler, president of the Prussian Academy of Sciences, read the problem he proposed for the Prize of Mathematics to be given in 1750 :

'Deduce from new principles, as simple as possible, a theory to explain the resistance exerted on a body moving in a fluid, as a function of the body's velocity, shape and mass, and of the fluid's density and compressibility'.

Six mathematicians, including d'Alembert, sent a manuscript, but Euler was not satisfied with them and decided to postpone the prize to 1752.

> *Grimberg*, D'Alembert et les équations aux dérivées partielles en hydrodynamique, Thèse de Doctorat, Université de Paris VII, 1998



Leonhard Euler (1707-1783)

Jean Le Rond d'Alembert (1717-1783)







1752: d'Alembert's paradox

D'Alembert was upset and took back his manuscript of 1749, translated it into French and published it in 1752.



1749

E S S A I D'UNE NOUVELLE THEORIE DE LA RÉSISTANCE DES FLUIDES Par M. D'ALEM BERT, de l'Académie Royale de Sciences de Paris, de celle de Proffe, & de la Sociédé Royale de Londres,

A PARIS, Chez DAVID l'ainé, Libraire, rue S. Jacques, à la Plume d'or. MDCCLII. AVEC APPROBATION ET PRIVILEGE DU ROL 1752

'It seems to me that the theory, developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance, a singular paradox which I leave to future geometers to elucidate.'

https://gallica.bnf.fr/ark:/12148/bpt6k206036b



Adhémar Jean-Claude Barré de Saint-Venant (1797-1886)



George Stokes (1819-1903)





Toshio Kato (1917-1999)



1904: Prandtl's boundary layer theory

- Prandtl (1904) predicted that the thickness of the boundary layer in contact with a solid body *(left)* scales as *Re^{-1/2}*, the inverse square root of the Reynolds number *Re*,
- But Prandtl's theory does not apply for separated flow regions where the boundary layer detaches from the solid body *(right)*.



Prandtl, Über Flüssigkeitsbewegung bei sehr kleiner Reibung, Proceedings of the 3rd ICM in Heidelberg, 484-491, 1904



What is the inviscid limit of Navier-Stokes?

Navier-Stokes equations with no-slip boundary conditions:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} & \text{for} \\ \nabla \cdot \mathbf{u} = 0 & \longrightarrow & \mathbf{u}_{\text{Re}}(t, \mathbf{X}) & \frac{\nu \to 0}{\text{Re} \to +\infty} \end{cases}$$

Re = VLv^{-1} the Reynolds number

Same initial conditions

Euler equations with slip boundary conditions:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p & \text{for} \\ \nabla \cdot \mathbf{u} = 0 & \longrightarrow & \mathbf{u}(t, \mathbf{X}) & v = 0 \\ \mathbf{u}_{|\partial\Omega} \cdot \mathbf{n} = \mathbf{0}, & \mathbf{u}(0, \cdot) = \mathbf{v} \end{cases} \longrightarrow \quad \mathbf{u}(t, \mathbf{X}) \quad \substack{v = 0 \\ \operatorname{Re} = +\infty} \end{cases}$$

Laboratory experiments



Numerical experiments



Both laboratory and numerical experiments show that the dissipation rate of turbulent flows becomes independent of the fluid viscosity for large *Re*



1984: Kato's theorem

Navier-Stokes solution converges towards the Euler solution, if and only if, energy dissipation vanishes

$$\Delta E_{\mathrm{Re}}(0,T) = \mathrm{Re}^{-1} \int_{0}^{T} \mathrm{d}t \int_{\Omega} \mathrm{d}\mathbf{x} \left| \nabla \mathbf{u}(t,\mathbf{x}) \right|^{2} \underset{\nu \to 0}{\longrightarrow} 0,$$

and, if and only if, this happens in a boundary layer of thickness inversely proportional to the Reynolds number *Re*

Kato, 1984, Remarks on zero viscosity limit for non stationary Navier-Stokes flows with boundary, MSRI Berkeley



This requires using smaller resolution to compute high Reynolds flows than predicted by Prandtl's theory



Dissipation of energy in the inviscid limit

• In an incompressible flow (
$$\rho = 1$$
)

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \int \frac{\mathbf{u}^2}{2} = -\nu \int \omega^2 = -2\nu Z$$

• To dissipate energy, vorticity needs to be created and/or amplified, in such a way that $Z \sim \nu^{-1}$.

Possible vorticity distributions: $\omega \sim \nu^{-1/2}$ over O(1) area, $\omega \sim \nu^{-1}$ over $O(\nu)$ area.

with E energy, Z enstrophy,
 ν fluid kinematic viscosity,
 ω flow vorticity.





2D Flow inside a cylinder



Dipole crashing onto a plane wall

DNS Resolution N=8192²





Dipole crashing onto a wall in 2D

Resolution N=16384²

Navier-Stokes equations with volume penalization integrated using Fourier

Nguyen van yen, M. F. and Schneider, PRL, **106**(18), 2011

t=0.5



t=0.3



Energy dissipation

Energy dissipated when the dipole crashes onto the wall at increasing Reynolds numbers



Production of dissipative structures



Production of dissipative structures



Energy Dissipating Structures Produced by Walls in Two-Dimensional Flows at Vanishing Viscosity

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2013

PHYSICS OF FLUIDS 25, 093104 (2013)

The effect of slip length on vortex rebound from a rigid boundary

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Comparison Navier-Stokes and Euler-Prandtl



Prandtl equation coupled to Euler

Ansatz for the vorticity field as $\text{Re} \to \infty$: $\omega(x, y) = \omega_E(x, y) + \nu^{-1/2} \omega_P(x, \nu^{-1/2}y) + \omega_R(x, y)$ **Prandtl's variable : y_P = y / v^{1/2}**

$$\begin{aligned} \partial_t \omega_P + \nabla (\mathbf{u}_P \omega_P) &= \partial_{y_P}^2 \omega_P \\ \omega_P(x, y_P, 0) &= 0 \\ \psi_P(x, y_P, t) &= \int_0^{y_P} \mathrm{d}y'_P \int_0^{y'_P} \mathrm{d}y''_P \omega_P(x, y''_P, t) \\ \partial_{y_P} \omega_P(x, 0, t) &= -\partial_x p_E(x, 0, t), \end{aligned}$$

where p_E is the pressure calculated from ω_E which is the vorticity given by Euler equation



Comparison Navier-Stokes and Euler-Prandtl

Navier-Stokes solver

- Fourier in x and compact finite differences of 5th order with non-uniform grid in y.
- Third order Runge-Kutta in t.
- Periodic in x and no-slip boundary conditions in y.

Euler solver

- Fourier with hyperdissipation in x and y.
- Third order Runge-Kutta in t.
- Mirror-symmetry around y=0 to impose boundary conditions.

Prandtl solver

- Second order finite differences in x and y.
- Second order semi-implicit Runge-Kutta in t.
- Neumann boundary condition at y=0 when inverting.



Computational grid





















Euler

Euler

Navier-Stokes

Euler

Euler

Navier-Stokes

Euler

Navier-Stokes

Euler

Prandtl's singularity

Prandtl equation has well-known finite time singularity

Prandtl solution's blow-up at t_D=55.8

According to Kato's theorem, and since ω_1 remains bounded uniformly until t_D , we expect that $\mathbf{u}_{\nu} \xrightarrow[\nu \to 0]{} \mathbf{u}_0$ uniformly on $[0, t_D]$.

Evolution of analyticity strip

Show convergence!

Vorticity along the wall at $t=50 < t_D$

Vorticity along the wall at $t=54 < t_D$

Vorticity along the wall at $t=55 < t_D$

Vorticity along the wall at t=55.3 < t_D

Spectrum of the boundary vorticity

The Prandtl's solution behaves as k^{-3/2} for large k, consistent with the build-up of a jump singularity of vorticity along the wall, while Navier-Stokes develops a bump which spreads in k with Re.

Vorticity along the wall at $t=57 > t_D$

Vorticity along the wall at t=57.5 > t_D

Scaling from Re=7692 to 123075

We observe Prandtl's scaling in Re^{1/2} before $t_D \sim 55.8$ and Kato's scaling in Re after.

What about the von Karman log law?

In turbulent boundary layers the mean velocity profile satisfies

$$\langle U(y) \rangle \simeq \frac{U_{\tau}}{K_{\text{karman}}} \log\left(\frac{yU_{\tau}}{\nu}\right)$$

the so called 'log law', where

$$U_{\tau} = \sqrt{\nu \left\langle \left. \frac{\mathrm{d}U}{\mathrm{d}y} \right|_{y=0} \right\rangle}$$

is the friction velocity.

This shows that the bulk velocity and U_{T} have the same scaling with Re. This can be seen as a statistical signature of a boundary layer thickness Re⁻¹, which is consistent in some sense with the existence of a Kato layer.

T. von Karman, Uber laminare und turbulente Reibung. Z. ang. Math. Mech. 1 (4), 233{252,, 1921

Conclusions

- The Prandtl solution becomes singular at t_Dwhen BL detaches.
- The Navier-Stokes solution converges uniformly to the Euler solution before BL detaches, and ceases to converge after BL detaches.
- The detached BL has spatial scales as fine as Re⁻¹, whic are produced in different directions and not only parallel to the wall, while the attached BL is parallel to the wall and scales as Re^{-1/2}.
- The maximal vorticity of Navier-Stokes solution does not appear at the same location of the Prandtl singularity. This contradicts the picture of BL detachment seen as a local process coinciding with Prandtl singularity.

Conclusions

• The velocity gradient du/dy at the wall scales like Re, which can be seen as the statistical signature of the existence of a boundary layer of thickness Re in the neighborhood of the wall.

 Hence, the log-law, which is obtained from experimental results, is consistent with the existence of a Kato layer.
 This connection can be made in a phenomenological way without invoking the Kolmogorov scale and cascade.

• Our results may help in investigating rigorous foundations to the phenomenological theory of von Karman.

Open questions

Numerical results suggest that a new asymptotic description of the flow beyond the breakdown of the Prandtl regime is possible. Studying it might help to answer the following questions:

- Would Navier-Stokes solution looses smoothness after t_D?
 Would it converges to a weak singular dissipative solution of Euler's equation analog to dissipative shocks in Burgers solution?
- How can such a weak solution be approximated numerically?

This might lead to a new resolution of d'Alembert's paradox in terms of the production of weak singular dissipative structures due to the interaction of fully-developed turbulent flows with walls.

J. Leray, 1934 Sur le mouvement d'un fluide visqueux, Acta Mathematica, **63** C. de Lellis and L. Székzlyhidi, 2010 Archives Rational Mechanics and Analysis, **195**(1), 221-260

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