Energy dissipation caused by boundary layer instability at vanishing viscosity

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Freie Universität, Berlin,

International Congress of Mathematicians,
Rio de Janeiro, August 3rd 2018

What is the inviscid limit of Navier-Stokes?

Navier-Stokes equations with no-slip boundary conditions:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p \boxed{+ \frac{1}{\mathrm{Re}}\boldsymbol{\nabla}^2 \mathbf{u}} \\ \boldsymbol{\nabla}\cdot\mathbf{u} = 0 \\ \mathbf{u}_{|\partial\Omega} = \mathbf{0}, \qquad \mathbf{u}(0,\cdot) = \mathbf{v} \end{cases} \longrightarrow \mathbf{u}_{\mathrm{Re}}(t,\mathbf{x})$$

for $\nu \to 0$, $\mathrm{Re} \to +\infty$

Re = VLν^{-1} the Reynolds number

Same initial conditions

?

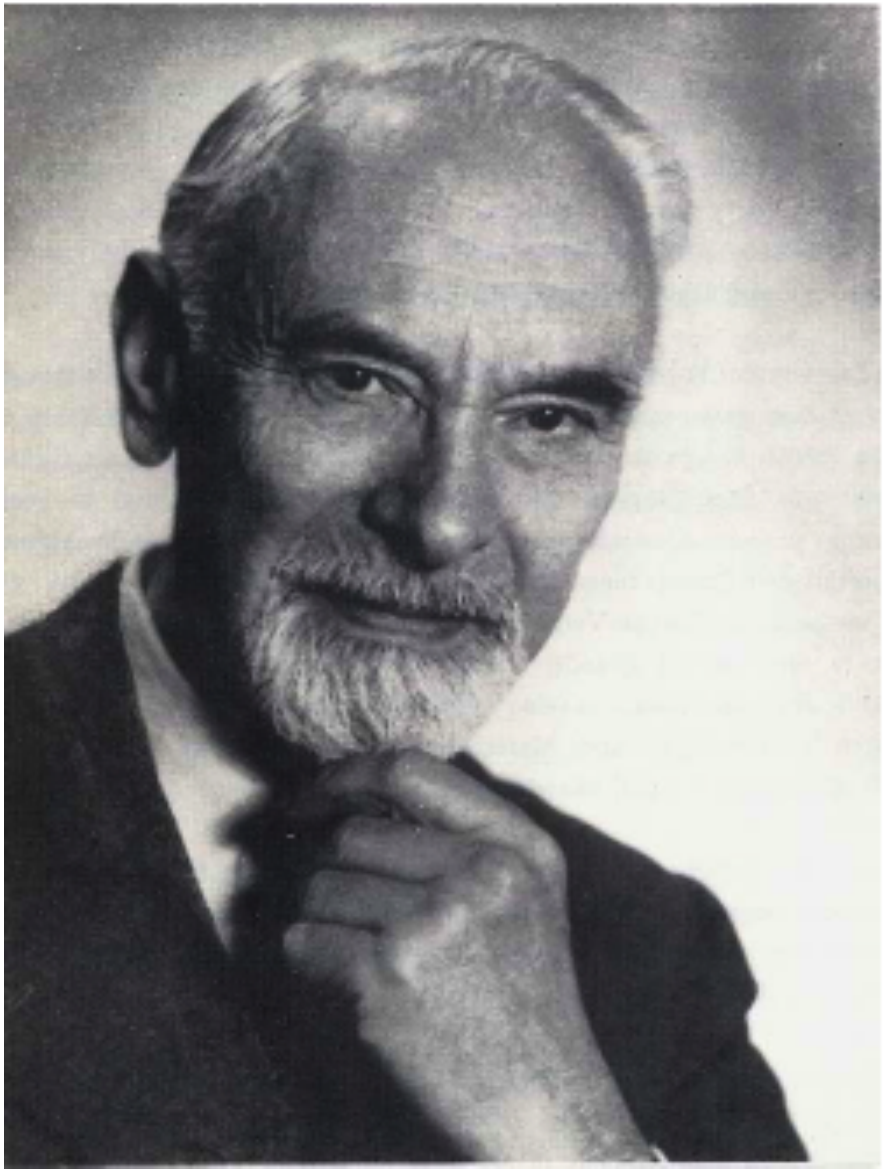
Euler equations with slip boundary conditions:

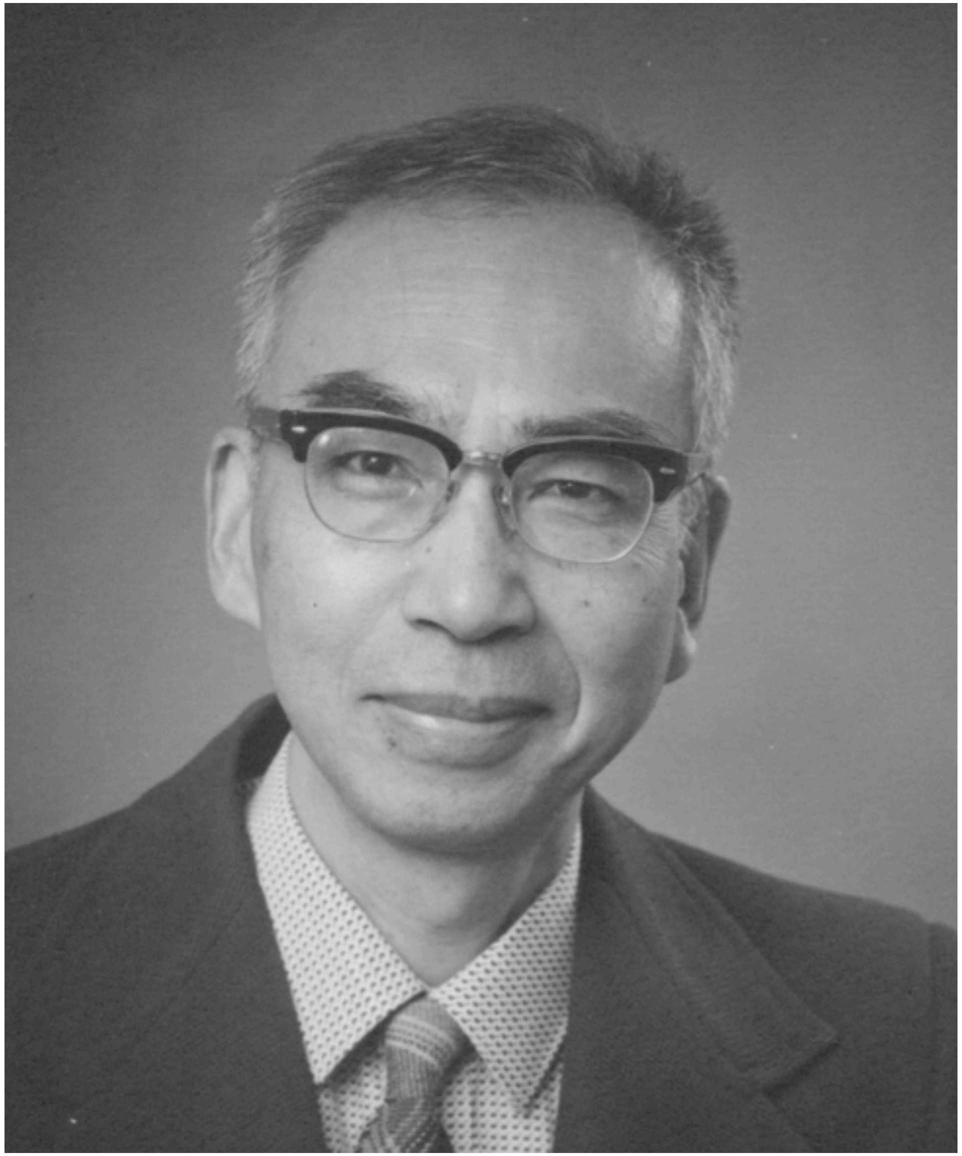
$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p \\ \boldsymbol{\nabla}\cdot\mathbf{u} = 0 \\ \mathbf{u}_{|\partial\Omega}\cdot\mathbf{n} = \mathbf{0}, \qquad \mathbf{u}(0,\cdot) = \mathbf{v} \end{cases} \longrightarrow \mathbf{u}(t,\mathbf{x})$$

for $\nu = 0$, $\mathrm{Re} = +\infty$

Ludwig Prandtl
(1875-1953)

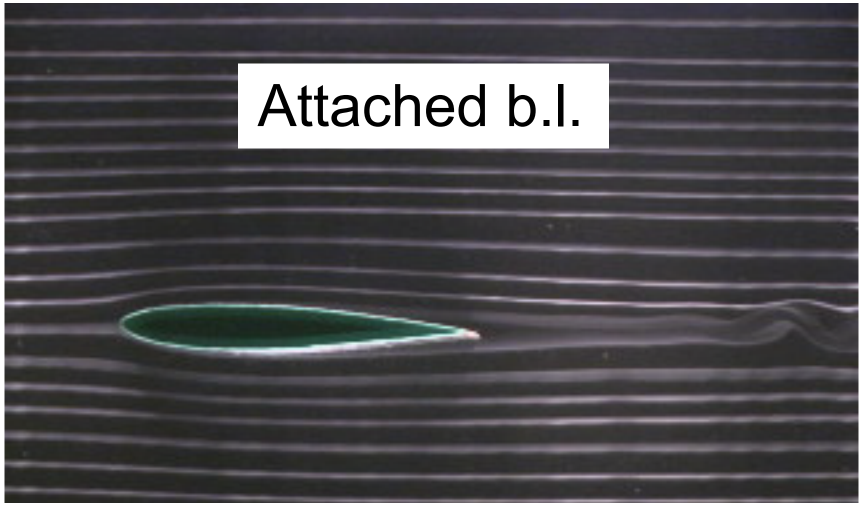
Toshio Kato
(1917-1999)

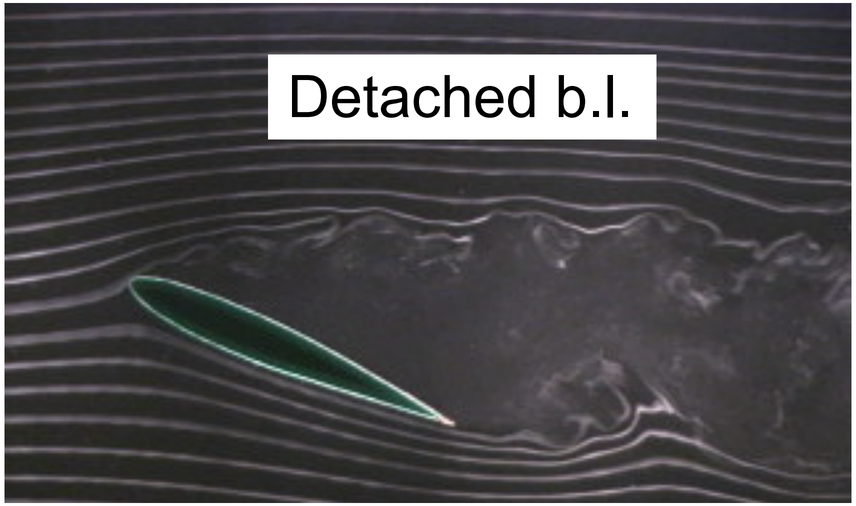




1904: Prandtl’s boundary layer theory

- Hypothesis: the fluid viscosity only plays a role in boundary layers in contact with no-slip walls, without any effect elsewhere.
- Prandtl (1904) predicted that the thickness of the viscous boundary layer scales as $Re^{-1/2}$, Re being the Reynolds number.
- But Prandtl’s theory does not apply to separated flow regions where the boundary layer detaches from the solid body.


Attached b.l.


Detached b.l.

Prandtl, Über Flüssigkeitsbewegung bei sehr kleiner Reibung, Proceedings of ICM in Heidelberg, 484-491, 1904

1984: Kato’s theorem

Navier-Stokes solution converges towards the Euler solution,
if and only if, energy dissipation vanishes

$$\Delta E_{\mathrm{Re}}(0,T) = \mathrm{Re}^{-1}\int_0^T \mathrm{d}t \int_\Omega \mathrm{d}\mathbf{x}\, \left|\nabla \mathbf{u}(t,\mathbf{x})\right|^2 \xrightarrow[\substack{\mathrm{Re}\to\infty \\ \nu \to 0}]{} 0,$$

and, if and only if, this happens in a boundary layer of
thickness inversely proportional to the Reynolds number *Re*

$$\xcancel{\delta x \propto \mathrm{Re}^{-\frac{1}{2}}}$$

$$\Big\Downarrow$$

$$\boxed{\delta x \propto \mathrm{Re}^{-1}}$$

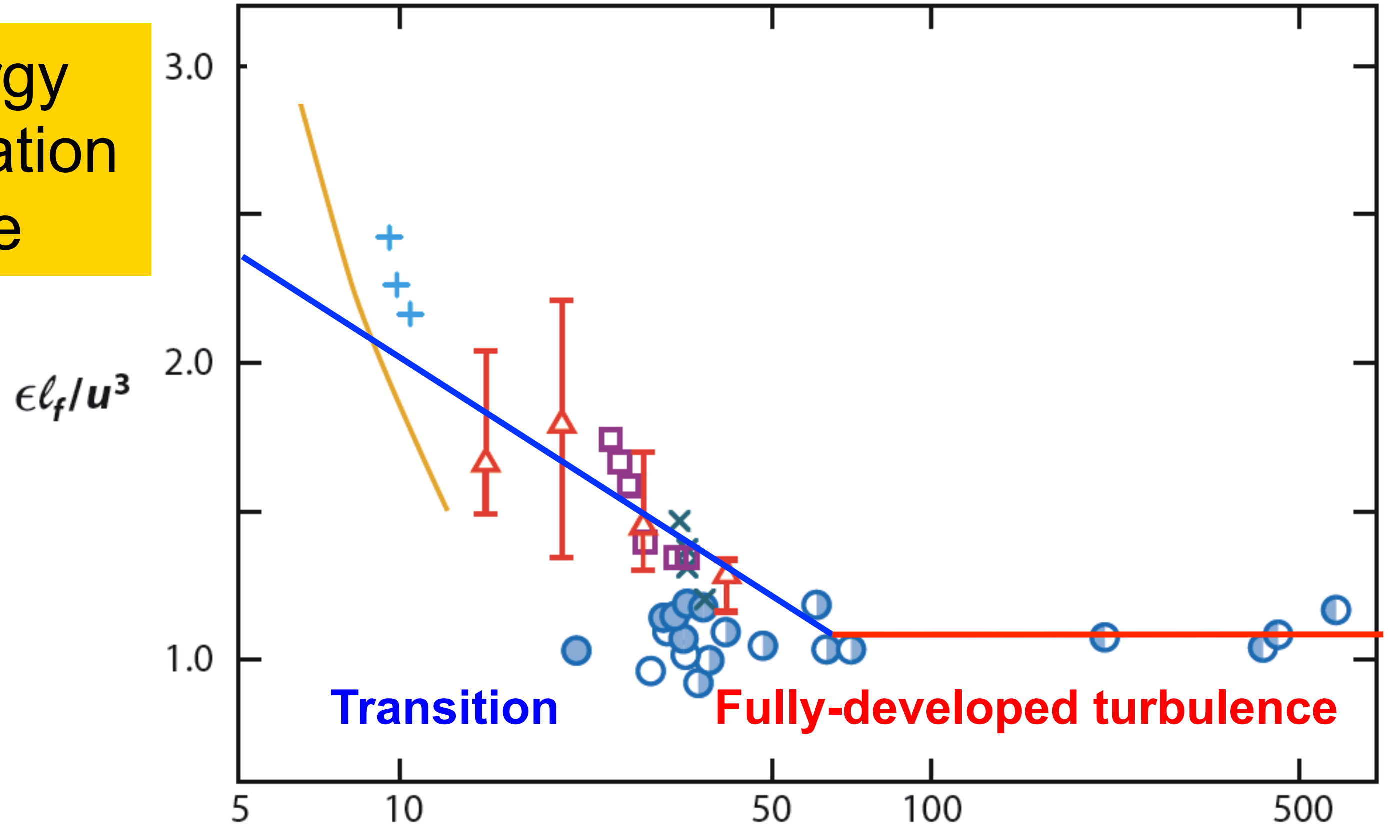
This requires using
smaller resolution
to compute high
Reynolds flows
than predicted by
Prandtl’s theory

Kato, 1984, Remarks on zero viscosity limit for non stationary Navier-Stokes flows with boundary, MSRI Berkeley

Laboratory experiments

Vassilicos, Ann. Rev. Fluid Mech., ***47****, 2015*

Energy dissipation rate



$Re_\lambda = Re^{1/2}$

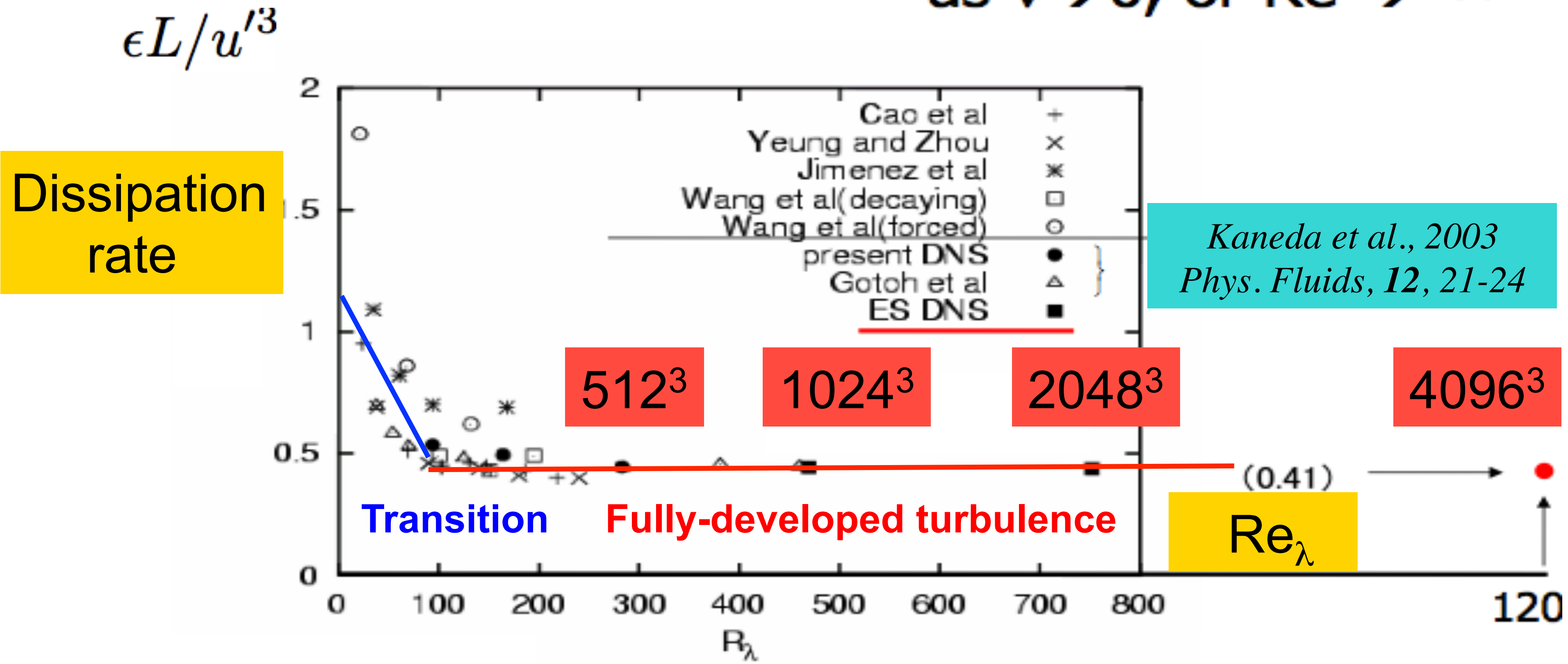
For $\nu \rightarrow 0$ or $\mathrm{Re} \rightarrow +\infty$

energy dissipation does not vanish but becomes constant

Numerical experiments

Normalized energy dissipation → ?

as ν→0, or Re → ∞



Kaneda et al., 2003
Phys. Fluids, ***12***, *21-24*

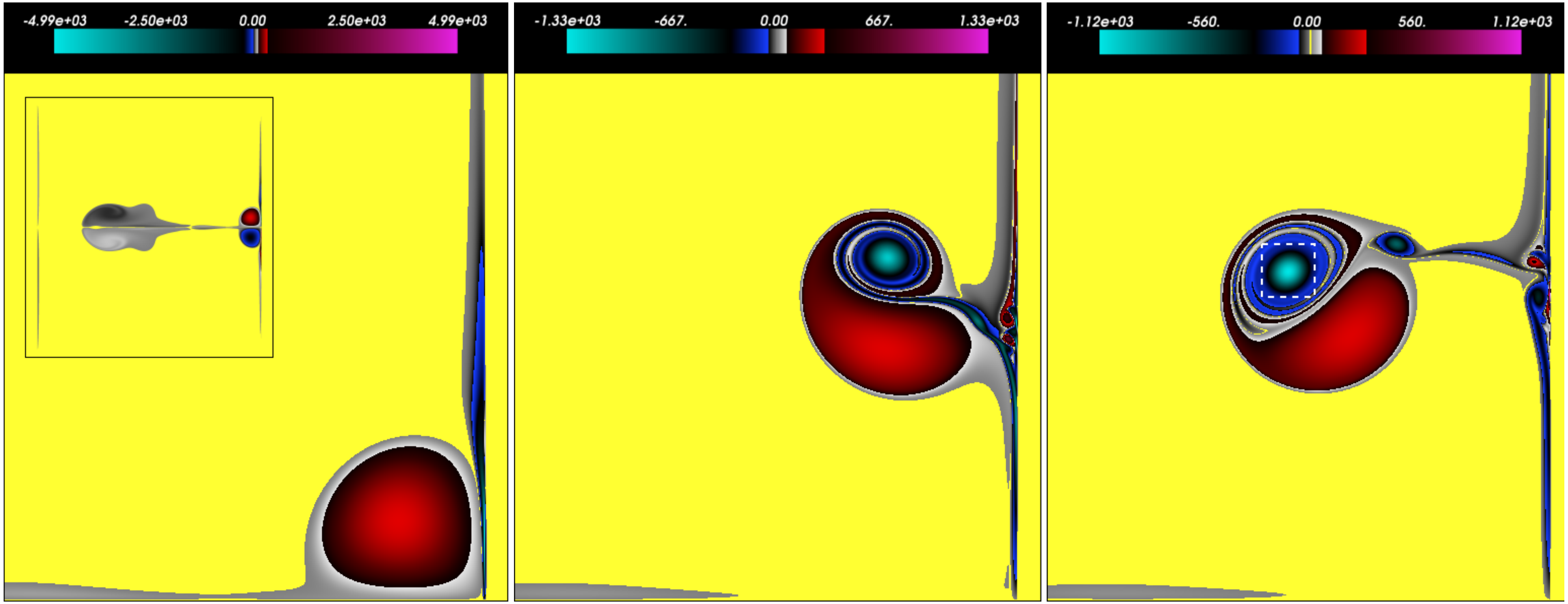
Both laboratory and numerical experiments
show that the dissipation rate of turbulent flows
becomes independent of the fluid viscosity for large *Re*

Dipole crashing onto a wall in 2D

Resolution
$N=16384^2$

Navier-Stokes equations
with volume penalization
integrated using Fourier

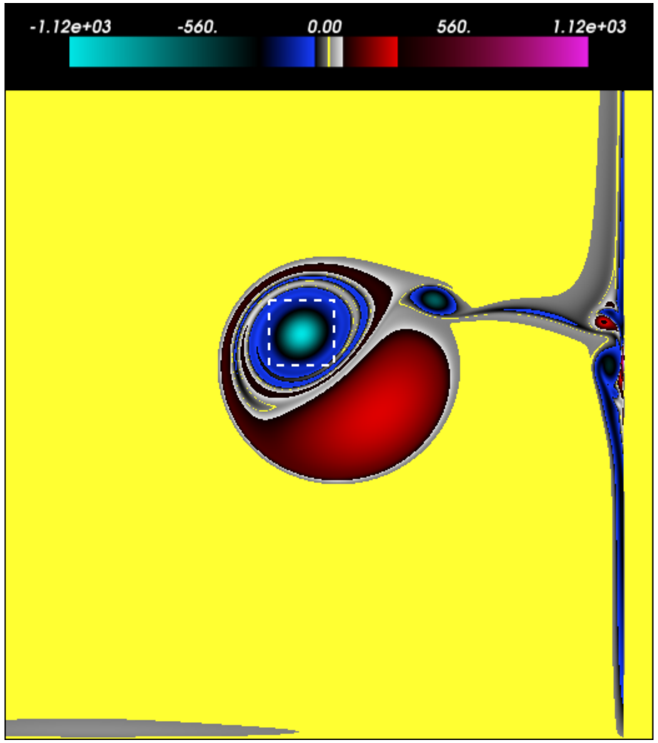
Nguyen van yen, M. F.
and Schneider,
*PRL, **106**(18), 2011*

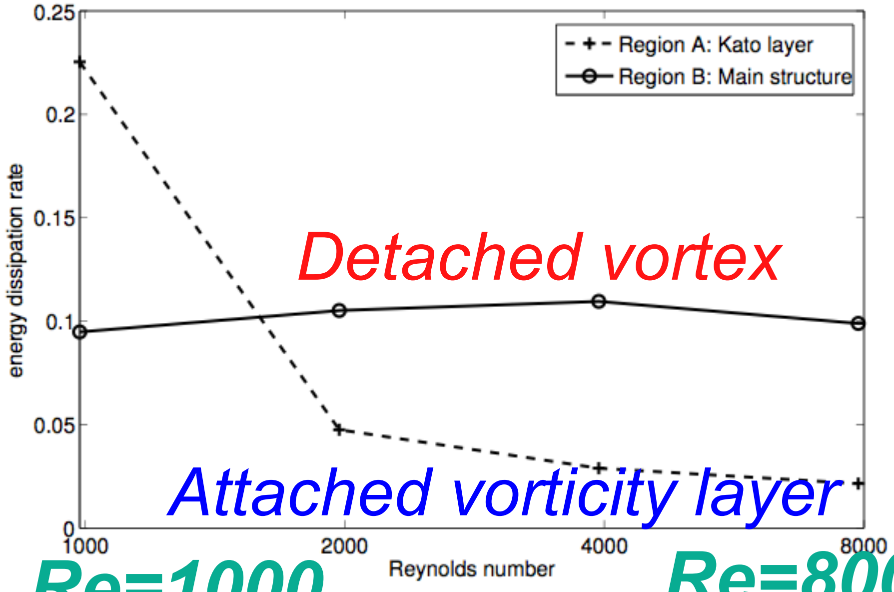


t=0.3   *t=0.4*   *t=0.5*

Production of dissipative structures

Nguyen van yen, M. F.
and Schneider,
PRL, ***106****(18), 2011*





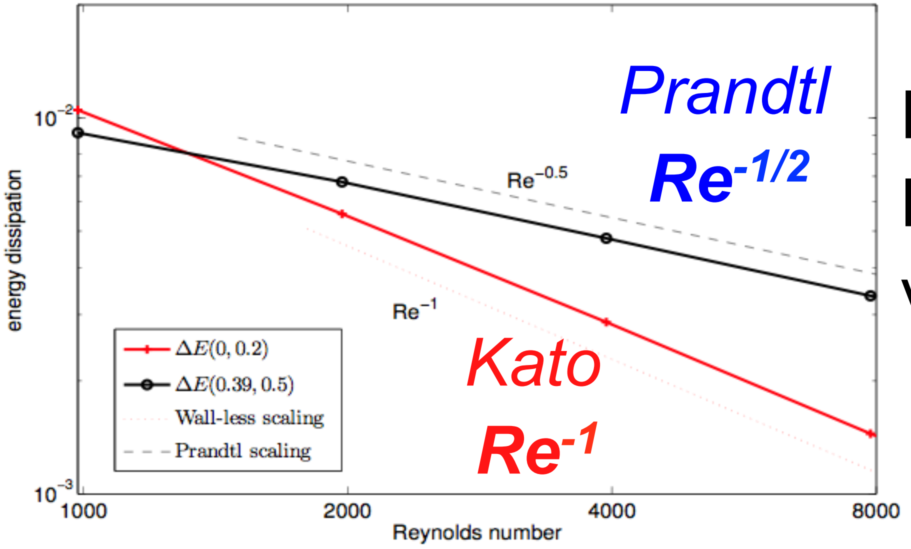
Detached vortex

Attached vorticity layer

Re=1000

Re=8000

Energy dissipation rate *(-2ν Z)* versus *Re*



Prandtl

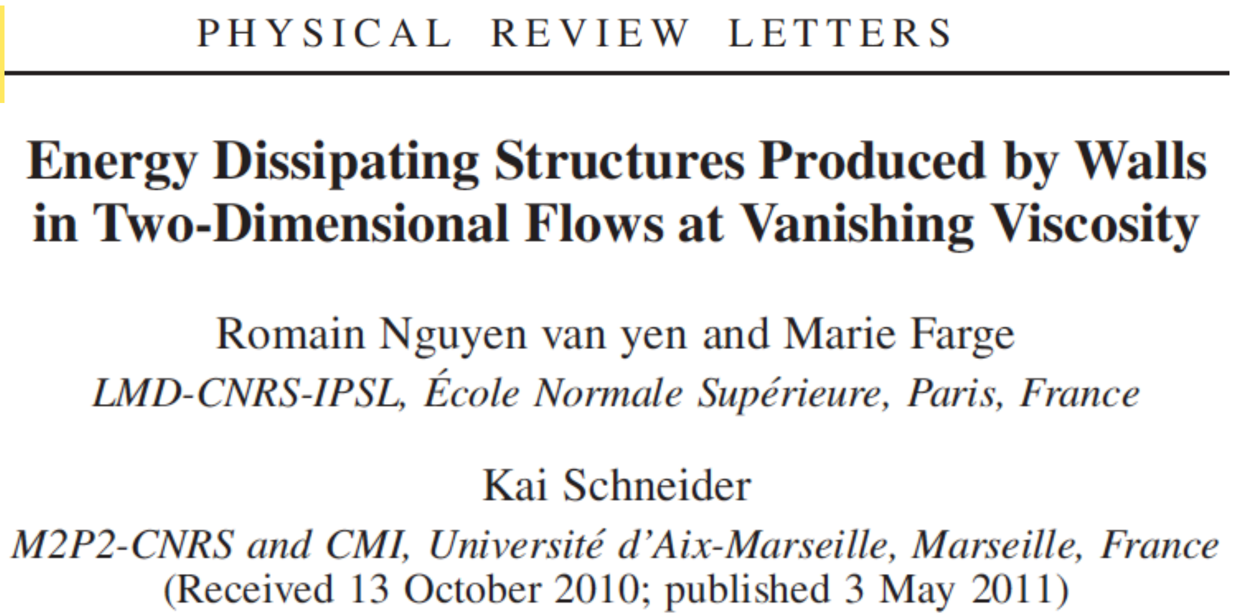
$Re^{-1/2}$

Kato

Re^{-1}

Energy Dissipation versus *Re*

2011



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Energy Dissipating Structures Produced by Walls in Two-Dimensional Flows at Vanishing Viscosity

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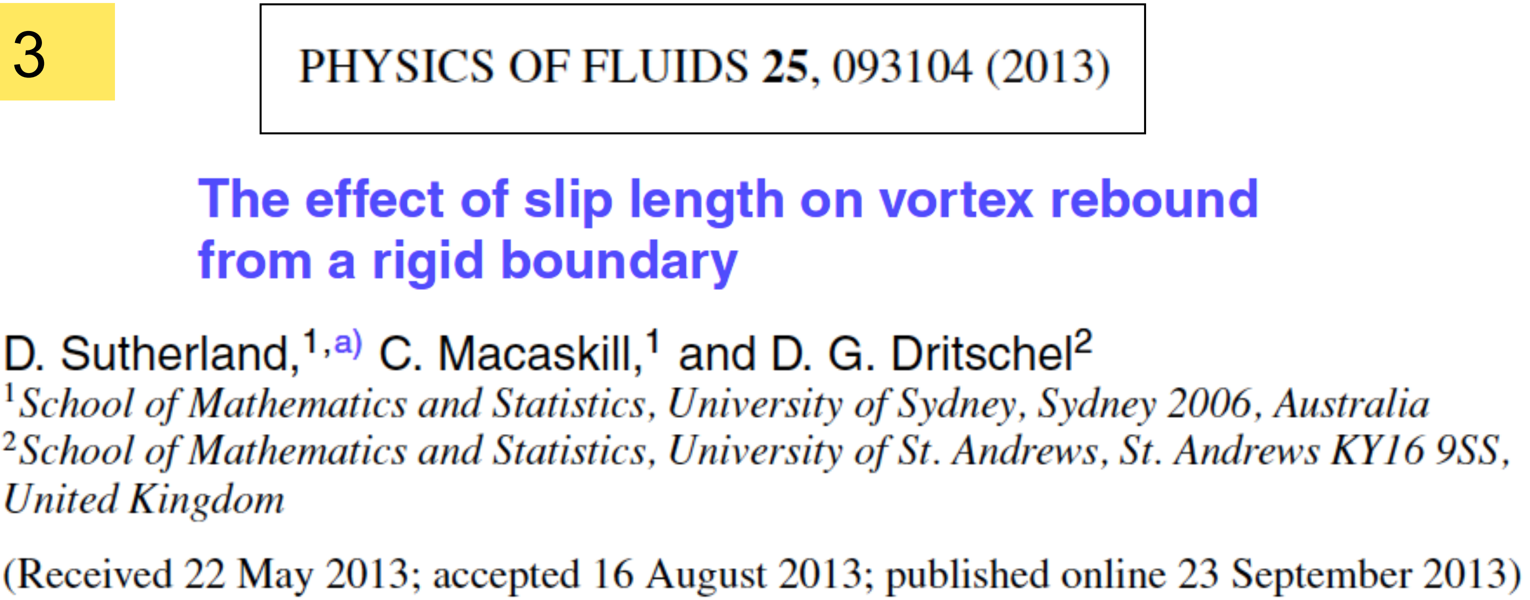
(Received 13 October 2010; published 3 May 2011)

2013



PHYSICS OF FLUIDS **25**, 093104 (2013)

The effect of slip length on vortex rebound from a rigid boundary

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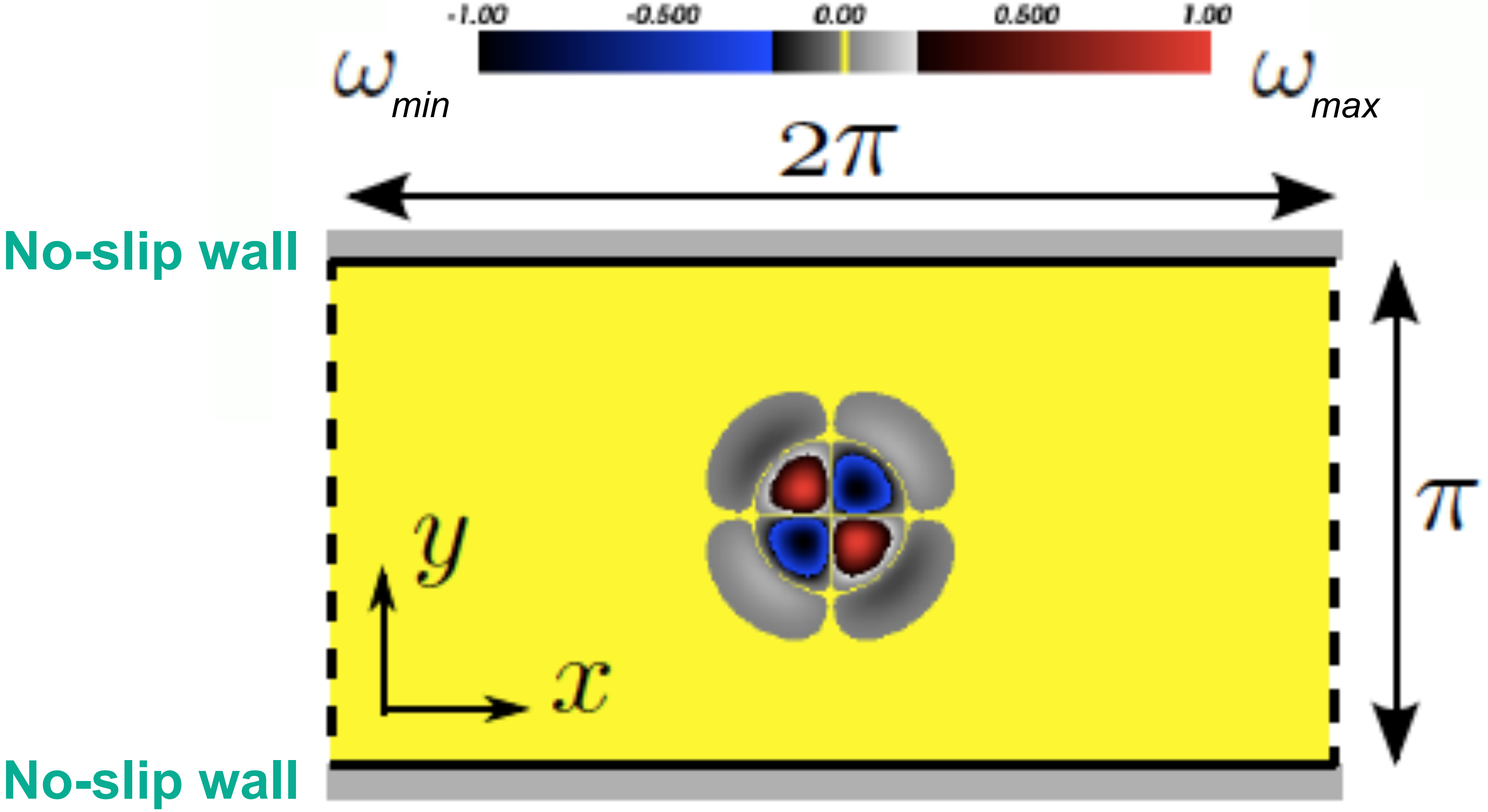
(Received 22 May 2013; accepted 16 August 2013; published online 23 September 2013)

Comparison Navier-Stokes and Euler-Prandtl

Initial vorticity field: vortex quadrupole



$$\psi_i(x,y) = Axy \exp\left(-\frac{(x-x_0)^2+(y-y_0)^2}{2s^2}\right)$$

Prandtl equation coupled to Euler

Ansatz for the vorticity field as $\mathrm{Re} \to \infty$:

$$\omega(x,y) = \omega_E(x,y) + \nu^{-1/2}\omega_P(x,\nu^{-1/2}y) + \omega_R(x,y)$$

Prandtl's variable : $y_P = y / \nu^{1/2}$

$$\boxed{\begin{aligned}
&\partial_t\omega_P + \boldsymbol{\nabla}.(\mathbf{u}_P\omega_P) = \partial^2_{y_P}\omega_P \\
&\omega_P(x,y_P,0) = 0 \\
&\psi_P(x,y_P,t) = \int_0^{y_P} \mathrm{d}y'_P \int_0^{y'_P} \mathrm{d}y''_P\omega_P(x,y''_P,t) \\
&\partial_{y_P}\omega_P(x,0,t) = -\partial_x p_E(x,0,t),
\end{aligned}}$$

where p_E is the pressure calculated from ω_E
which is the vorticity given by Euler equation

Numerical solvers

Navier-Stokes solver

- Fourier in x and compact finite differences of 5th order with non-uniform grid in y.
- Third order Runge-Kutta in t.
- Periodic in x and no-slip boundary conditions in y.

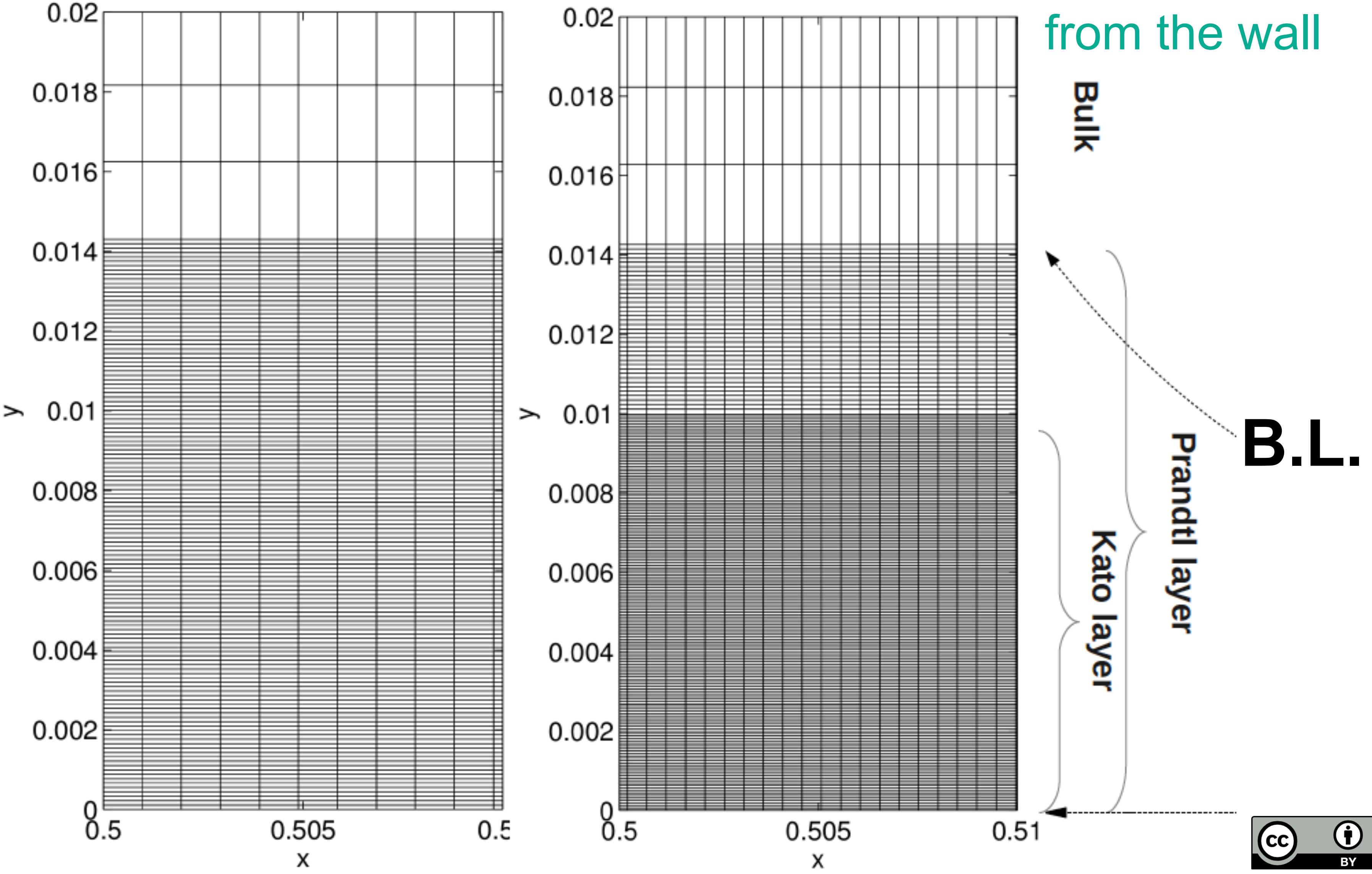
Euler solver

- Fourier with hyperdissipation in x and y.
- Third order Runge-Kutta in t.
- Mirror-symmetry around y=0.

Prandtl solver

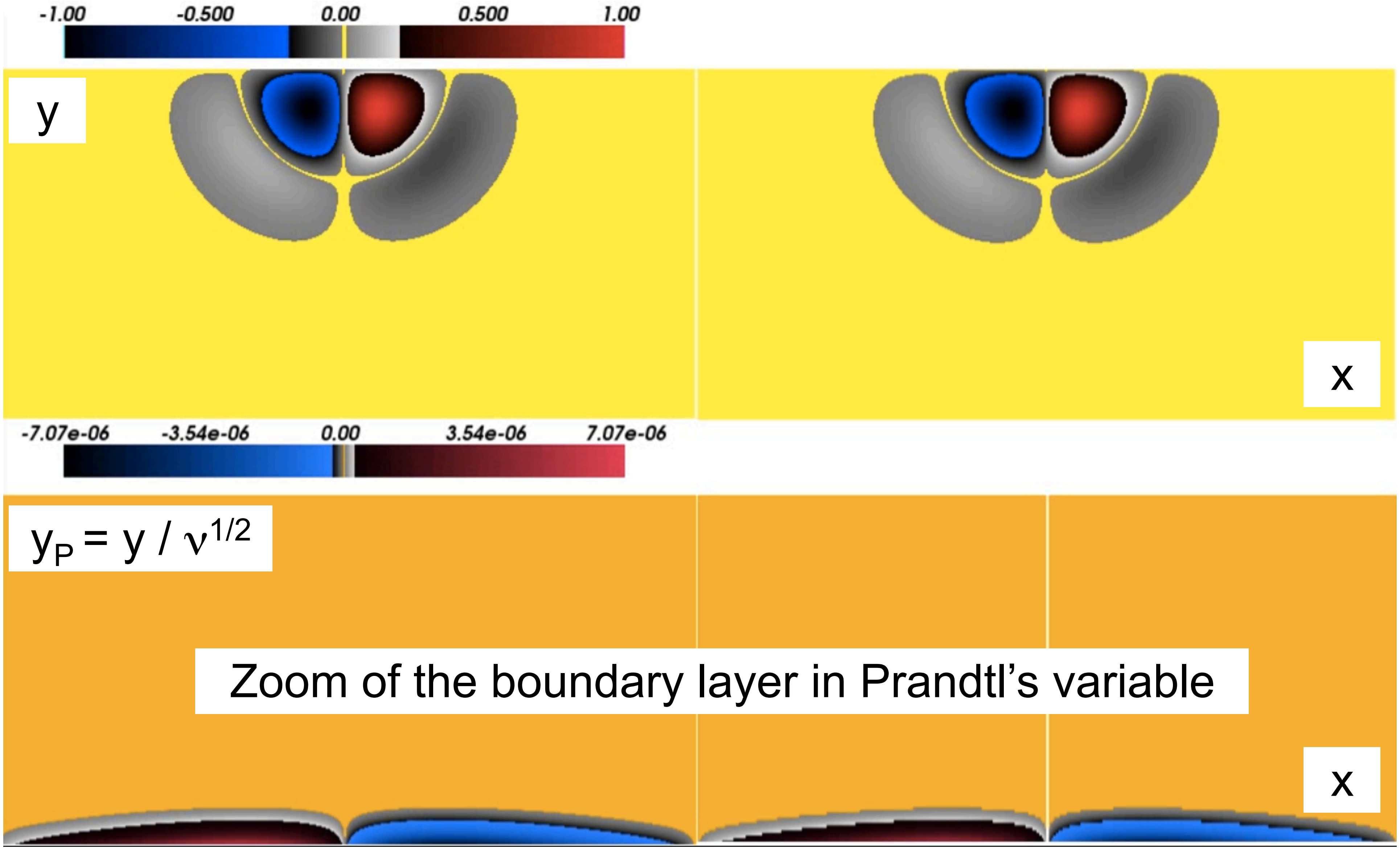
- Second order finite differences in x and y.
- Second order semi-implicit Runge-Kutta in t.
- Neumann boundary condition for vorticity at y=0.

Computational grid

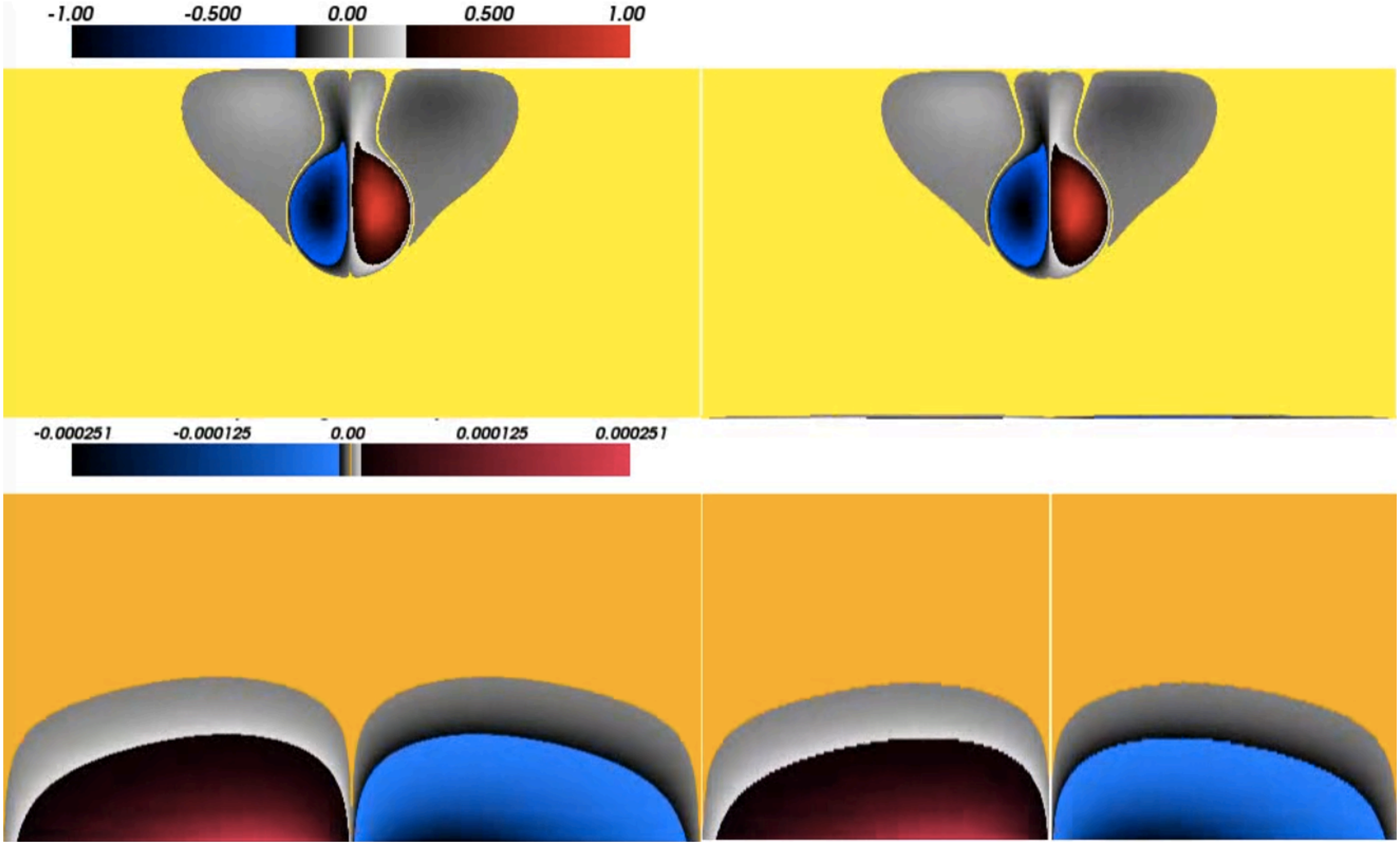


Before t=54

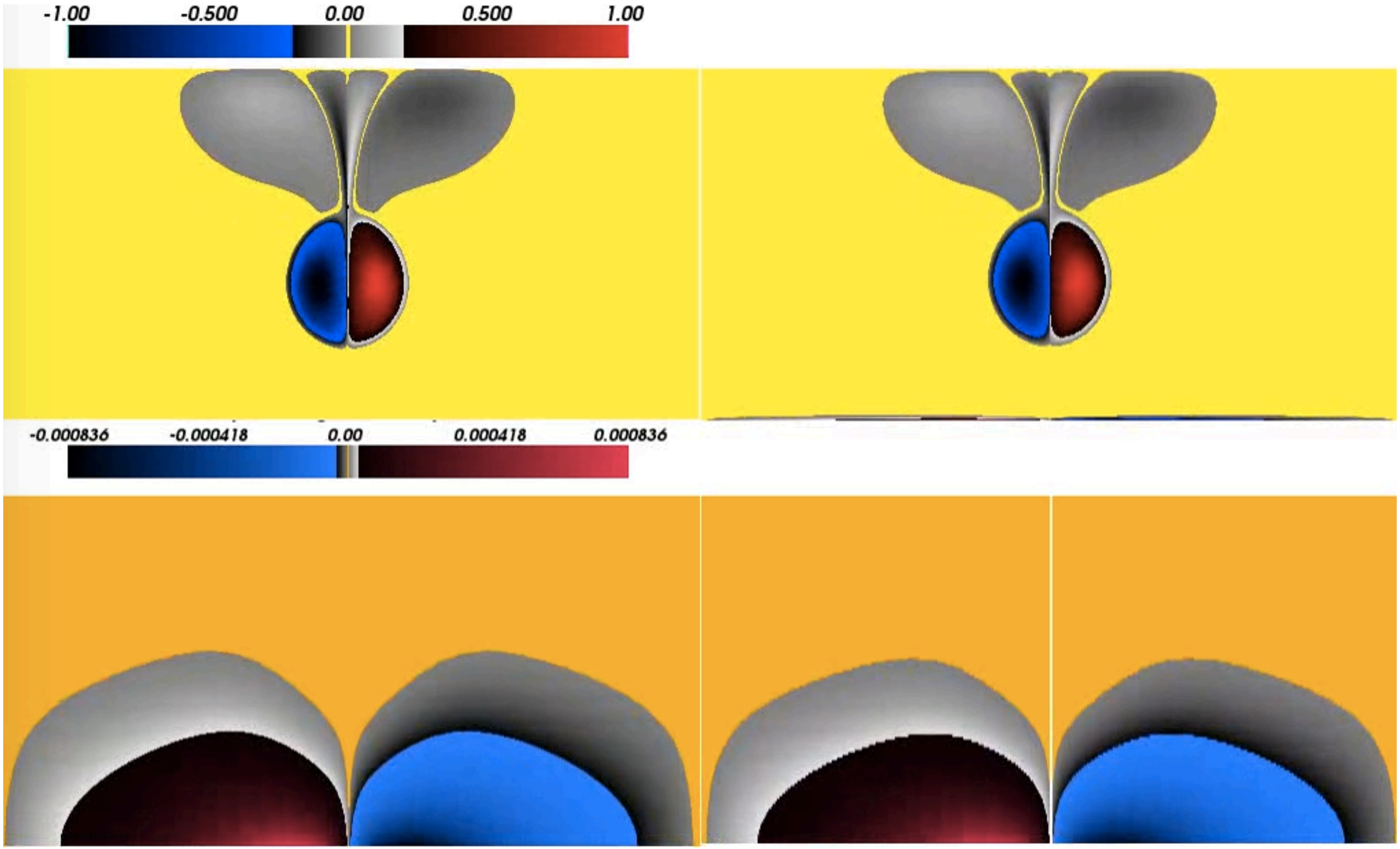
After t=54 when B.L. detaches from the wall



Euler and Prandtl **Navier-Stokes**



Euler and Prandtl **Navier-Stokes**



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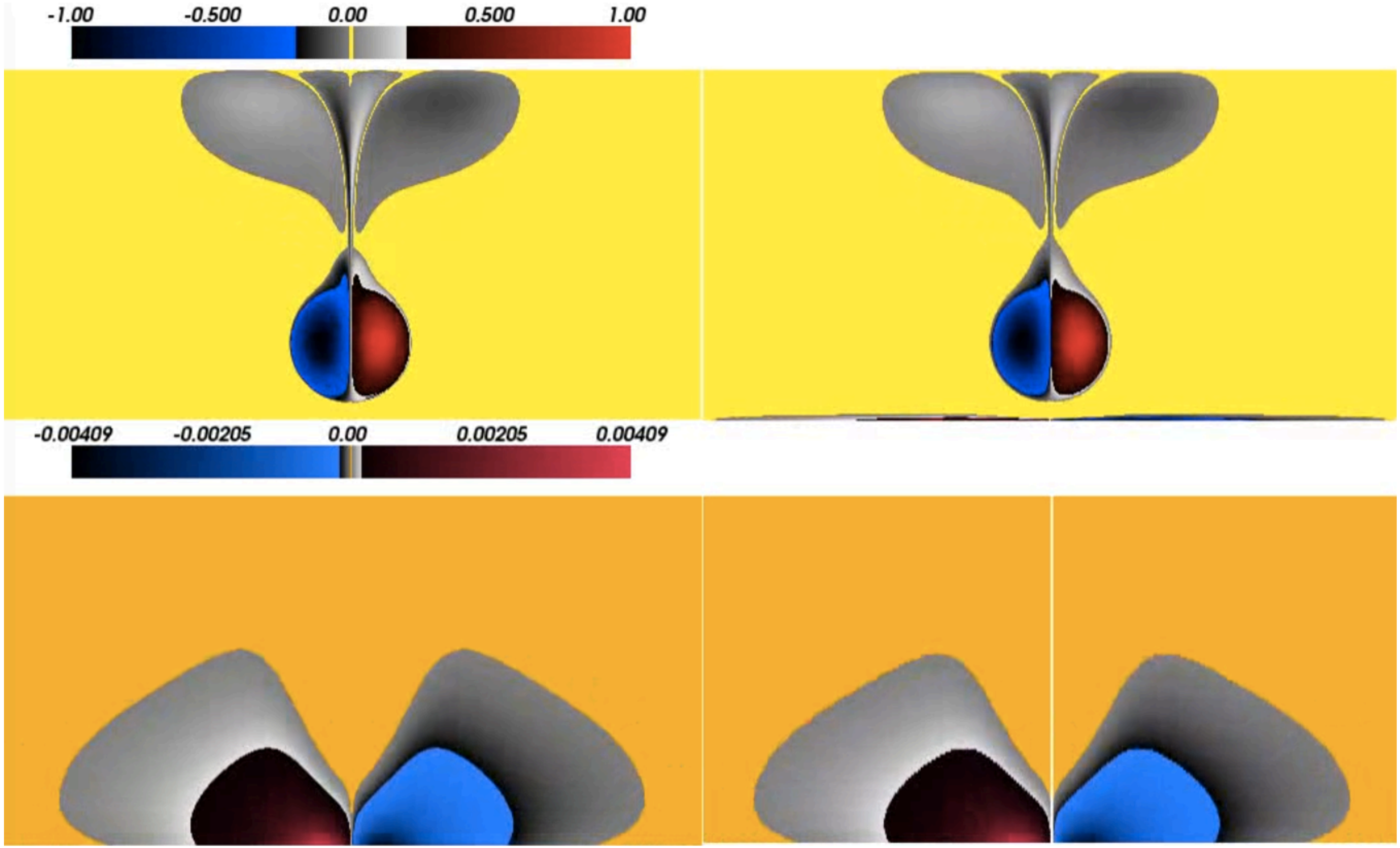
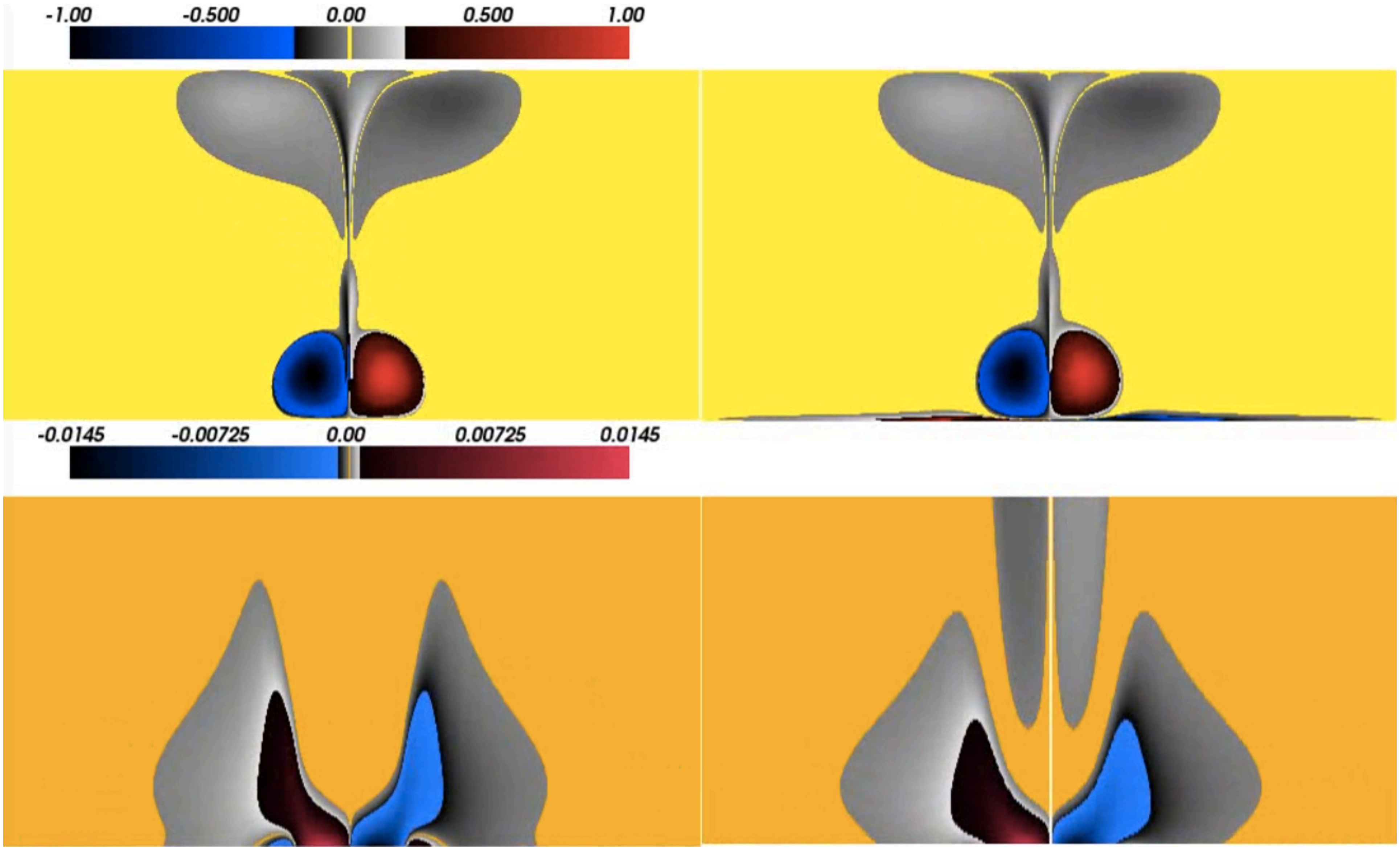
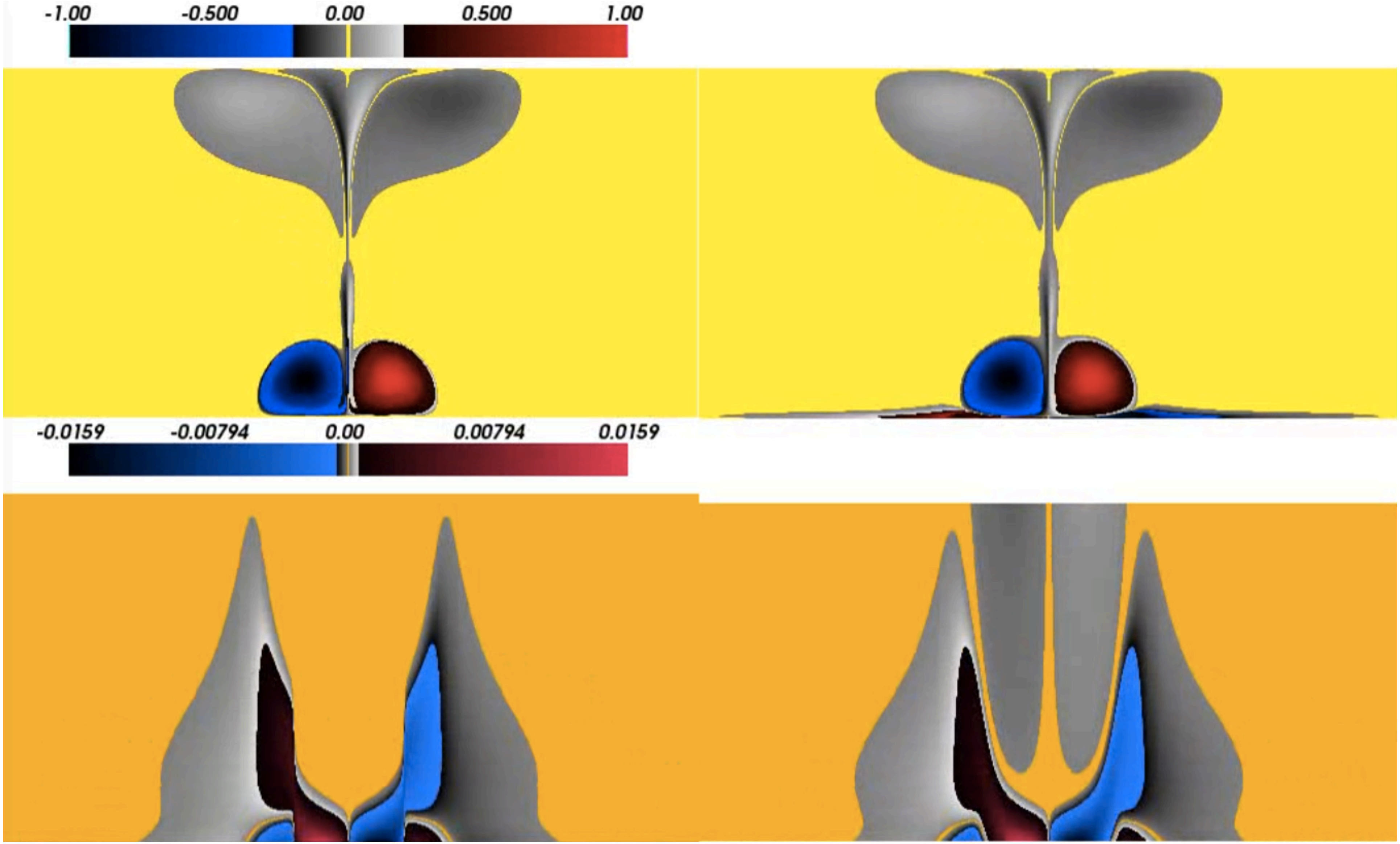


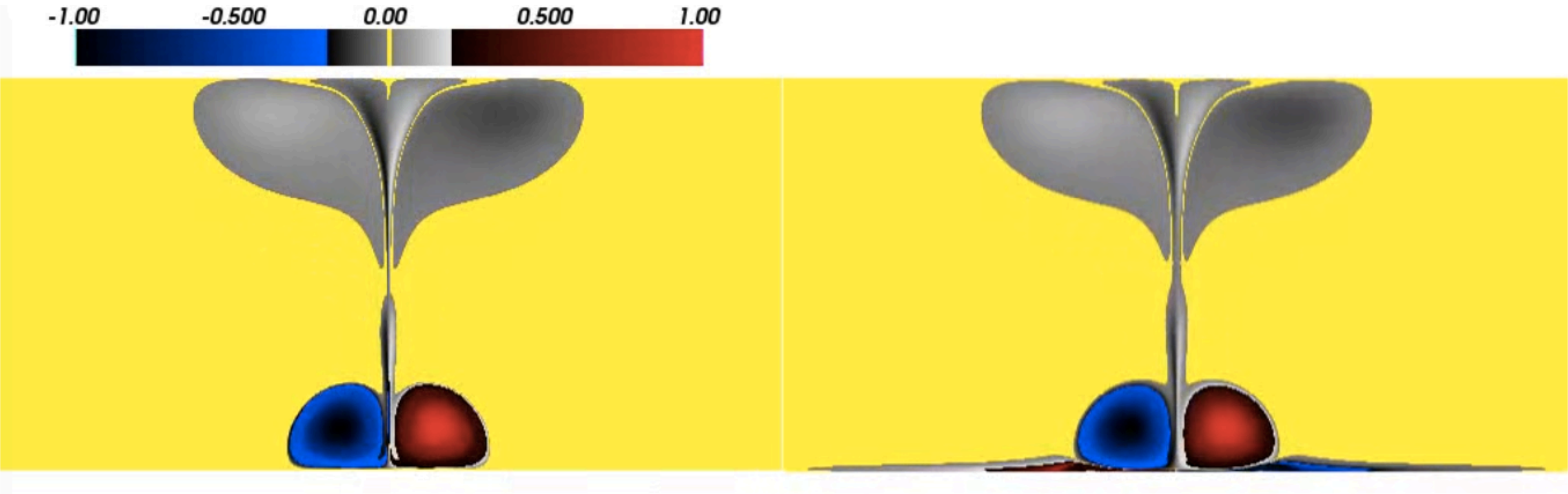
Euler and Prandtl **Navier-Stokes**



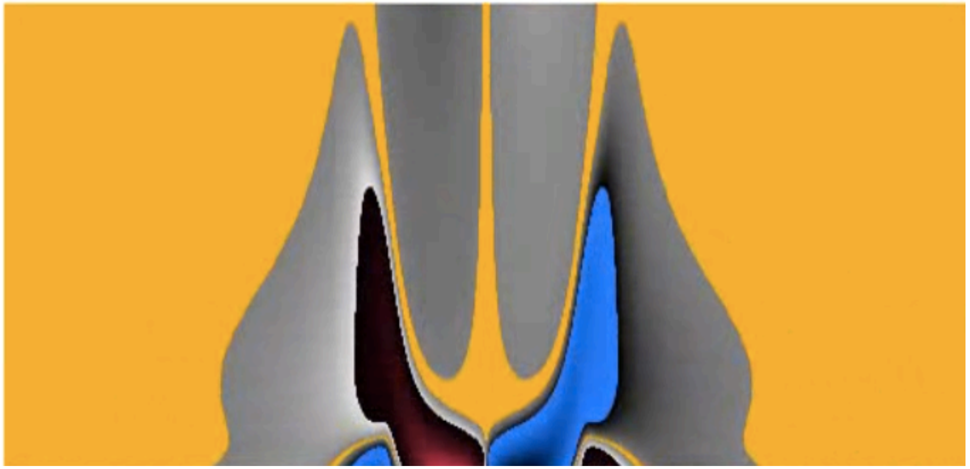
Euler and Prandtl **Navier-Stokes**



Euler and Prandtl **Navier-Stokes**

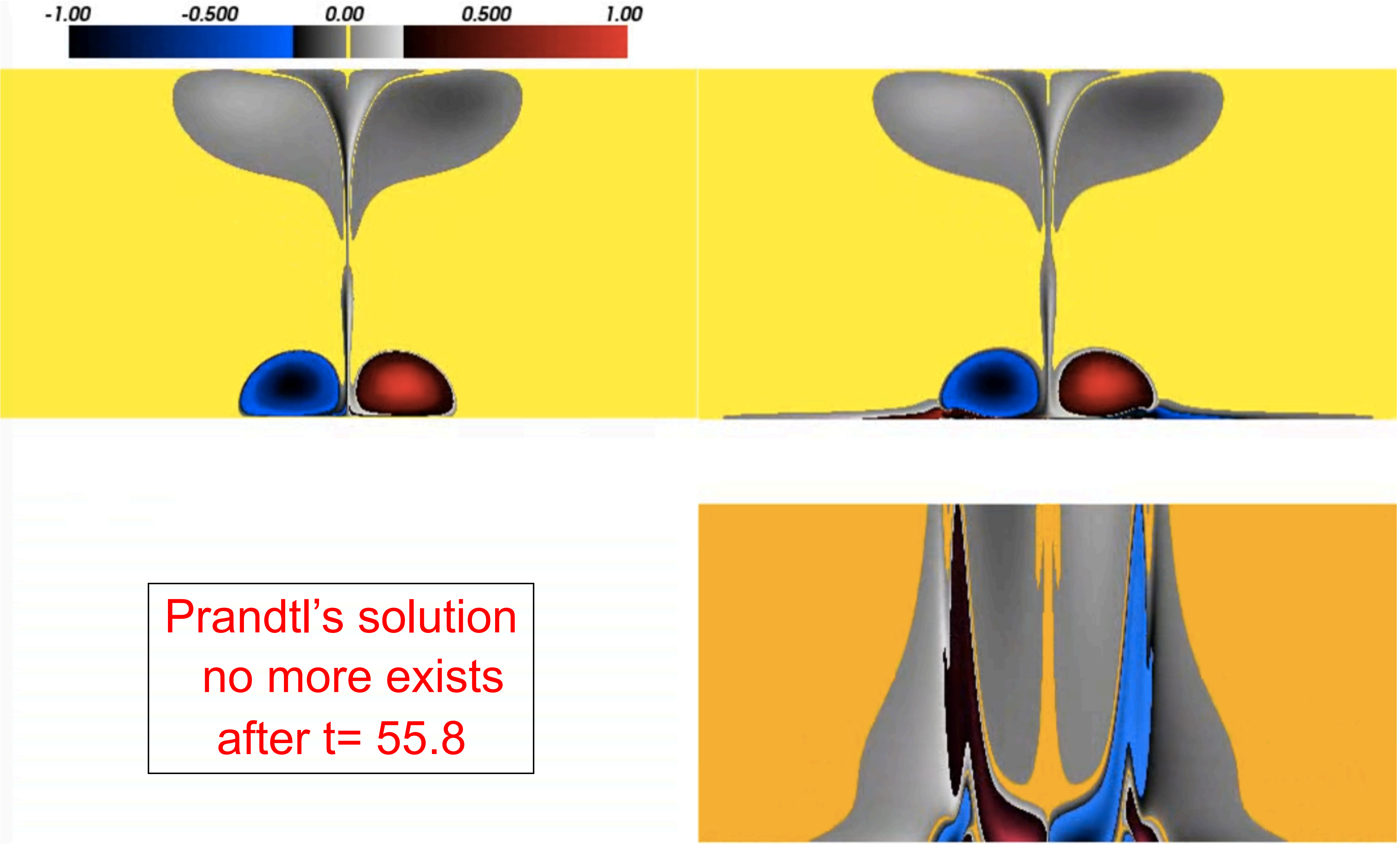


Prandtl's solution
no more exists
after t= 55.8



Euler

Navier-Stokes



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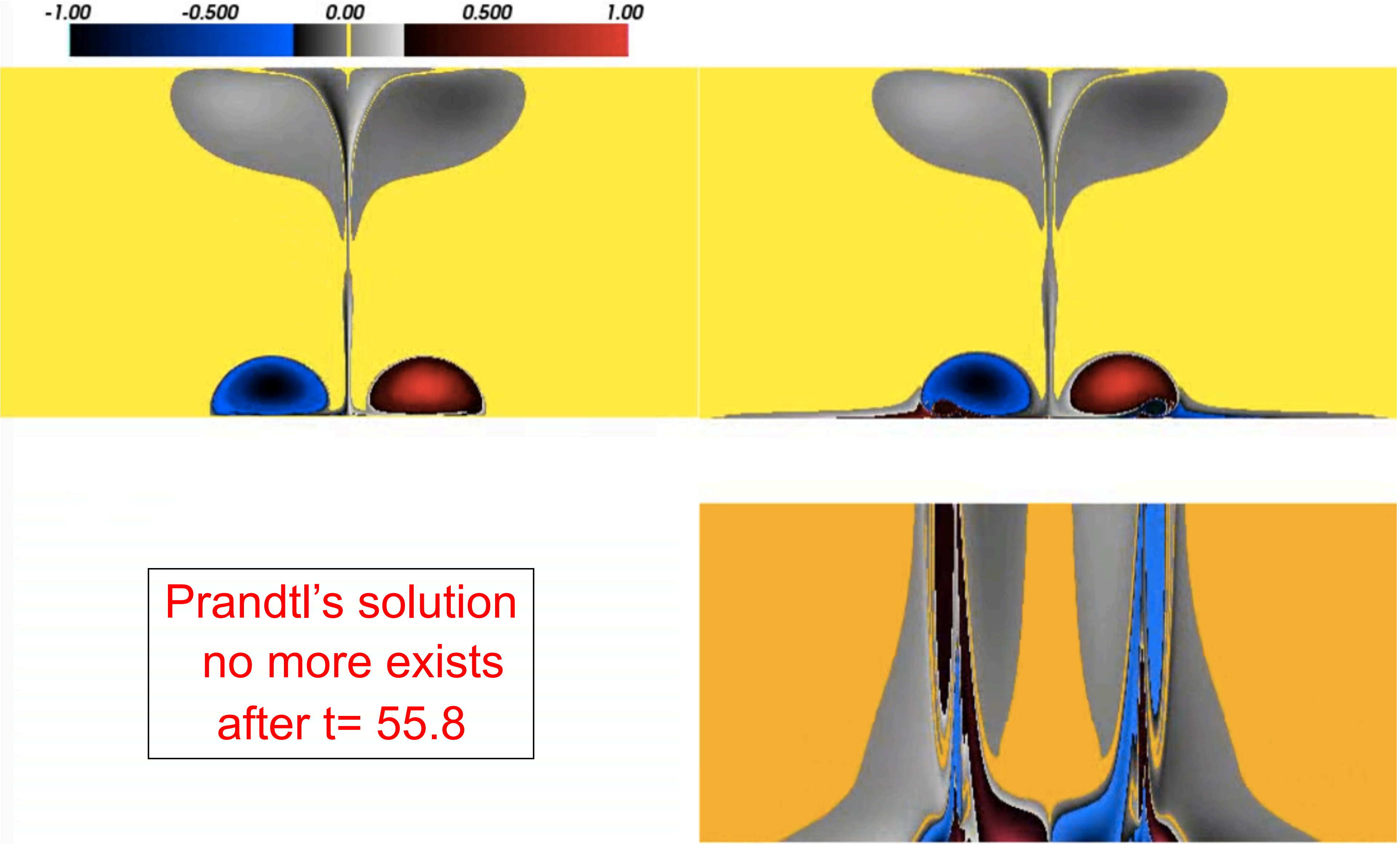
Euler

Navier-Stokes



Prandtl’s solution
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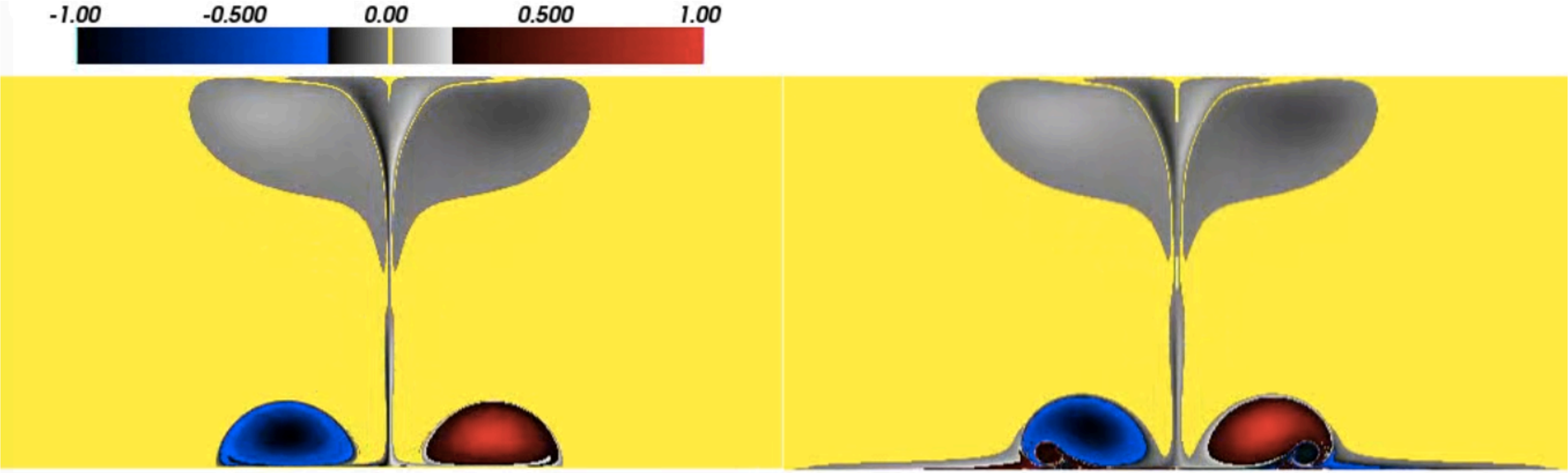
Euler

Navier-Stokes



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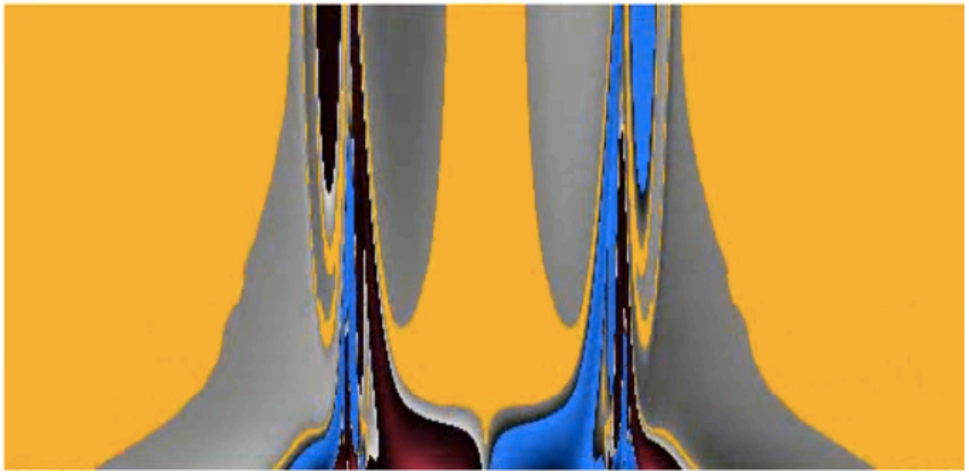
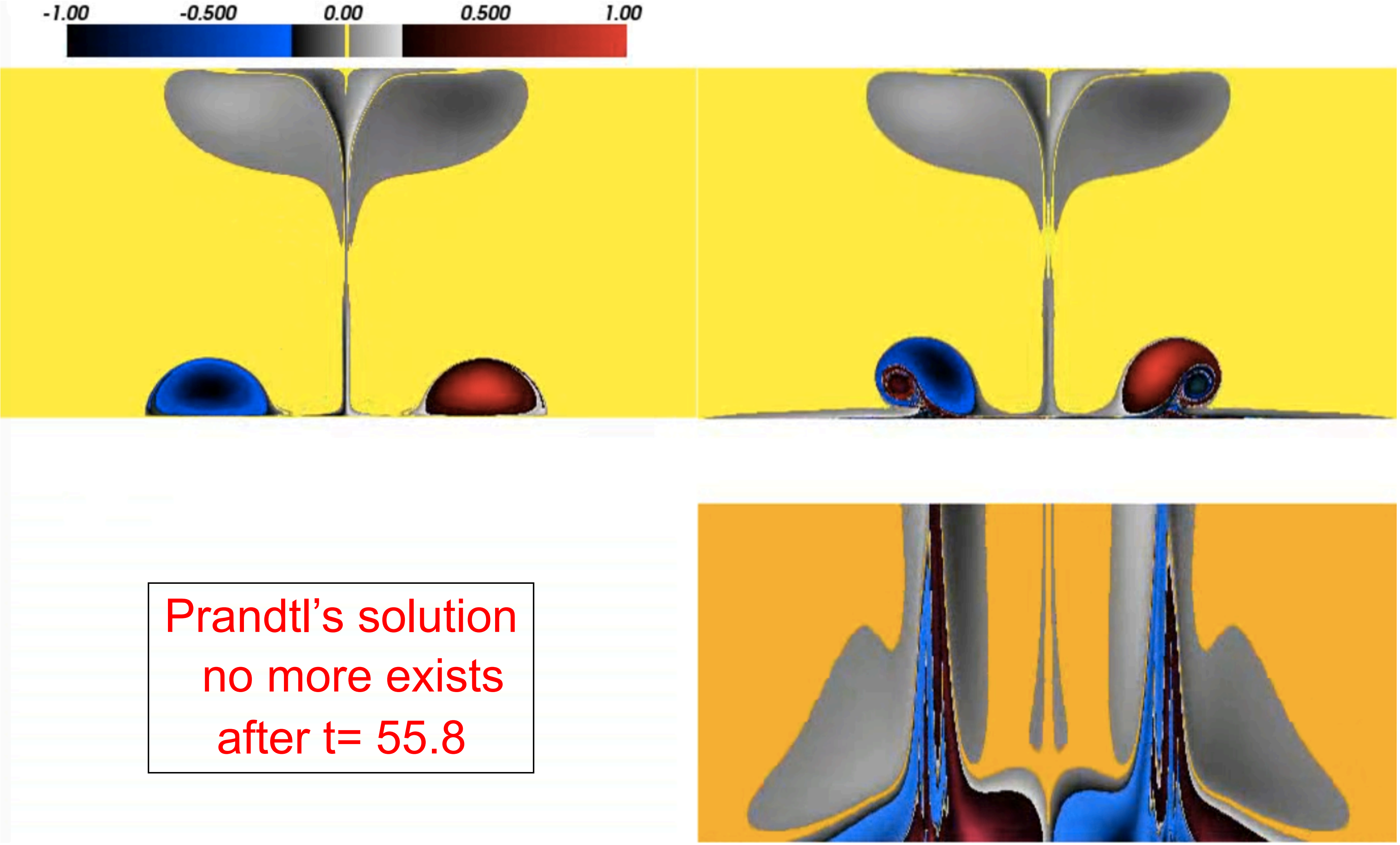


Euler

Navier-Stokes



Prandtl's solution
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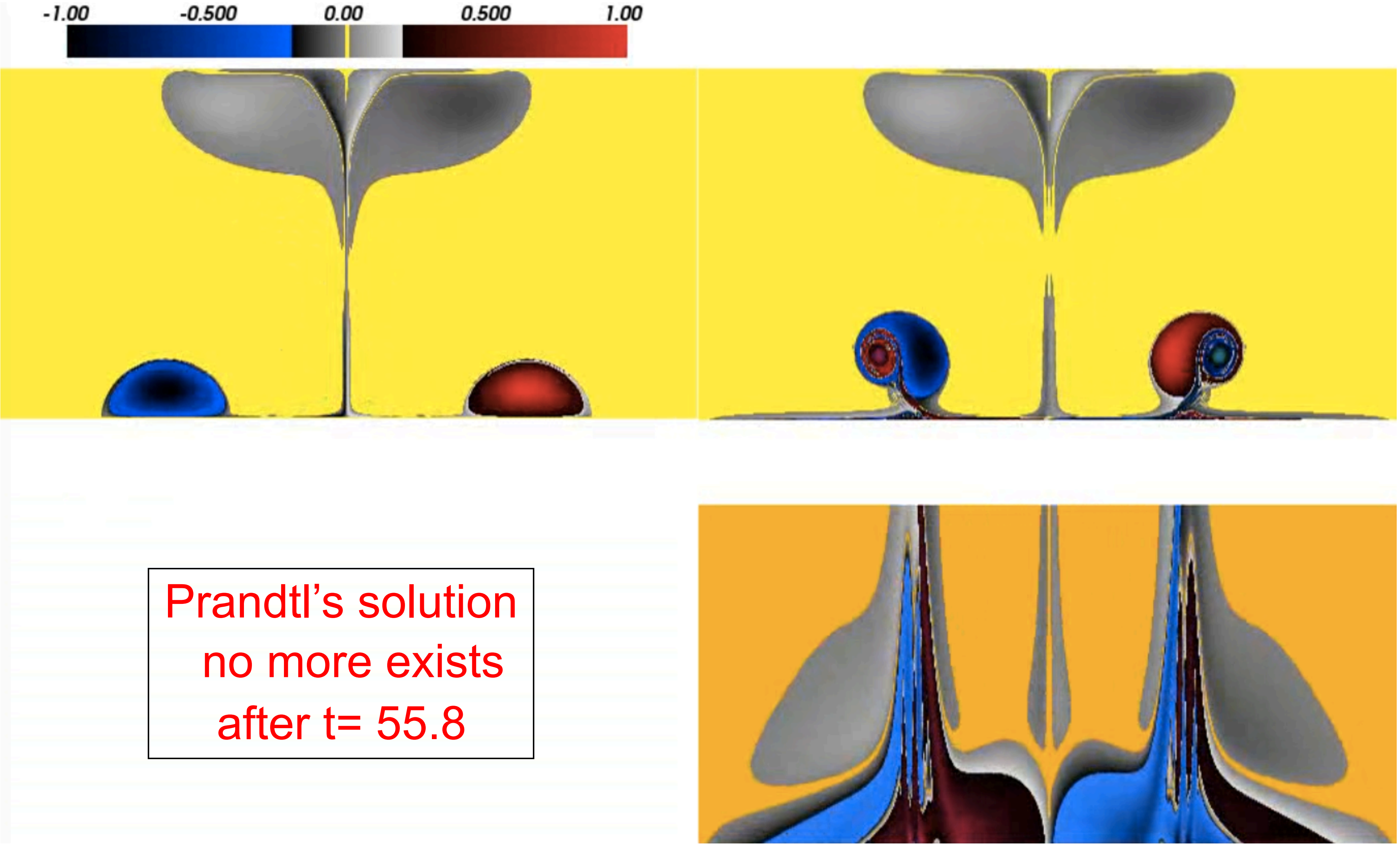
Euler

Navier-Stokes



Prandtl’s solution
no more exists
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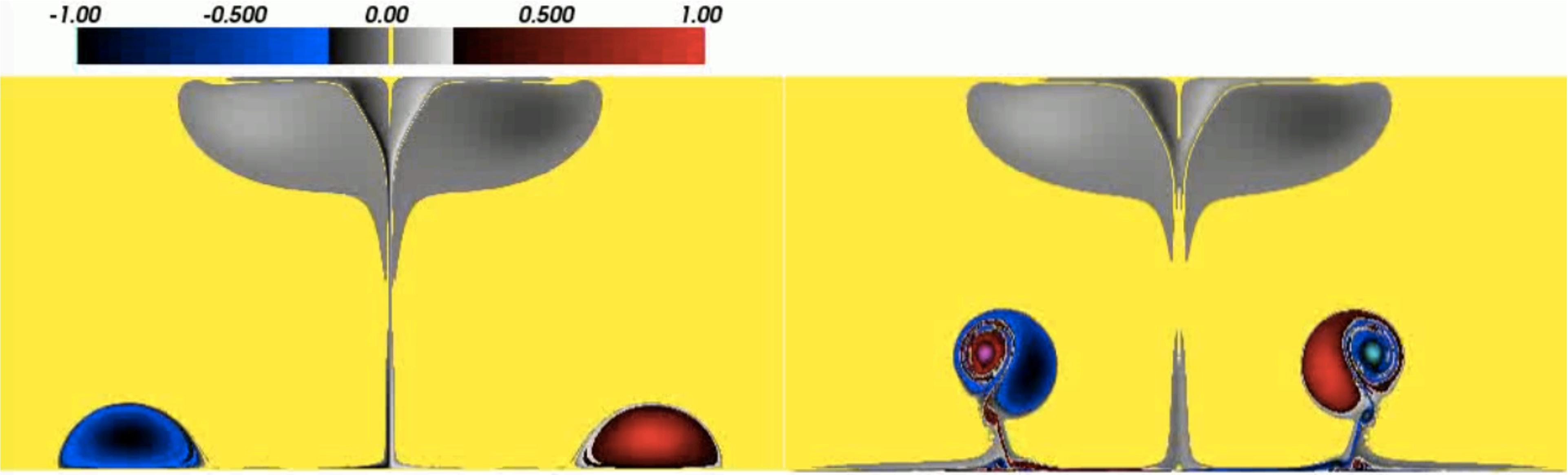
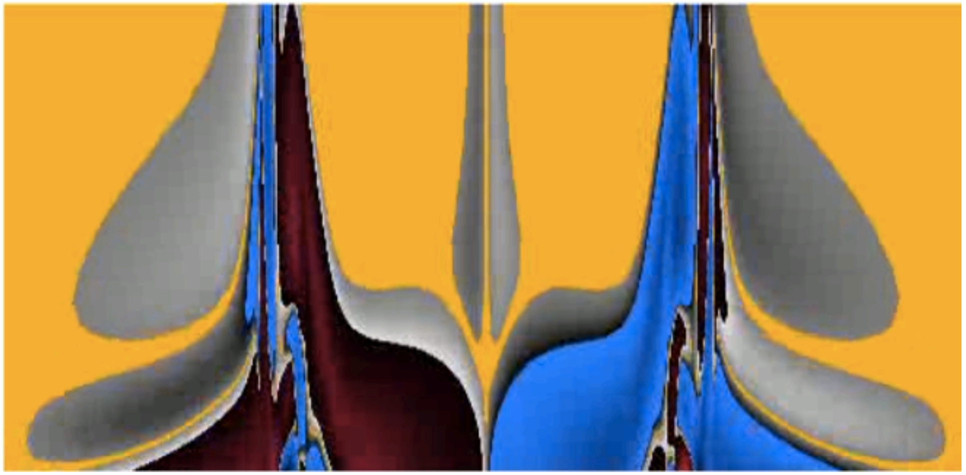
Euler

Navier-Stokes

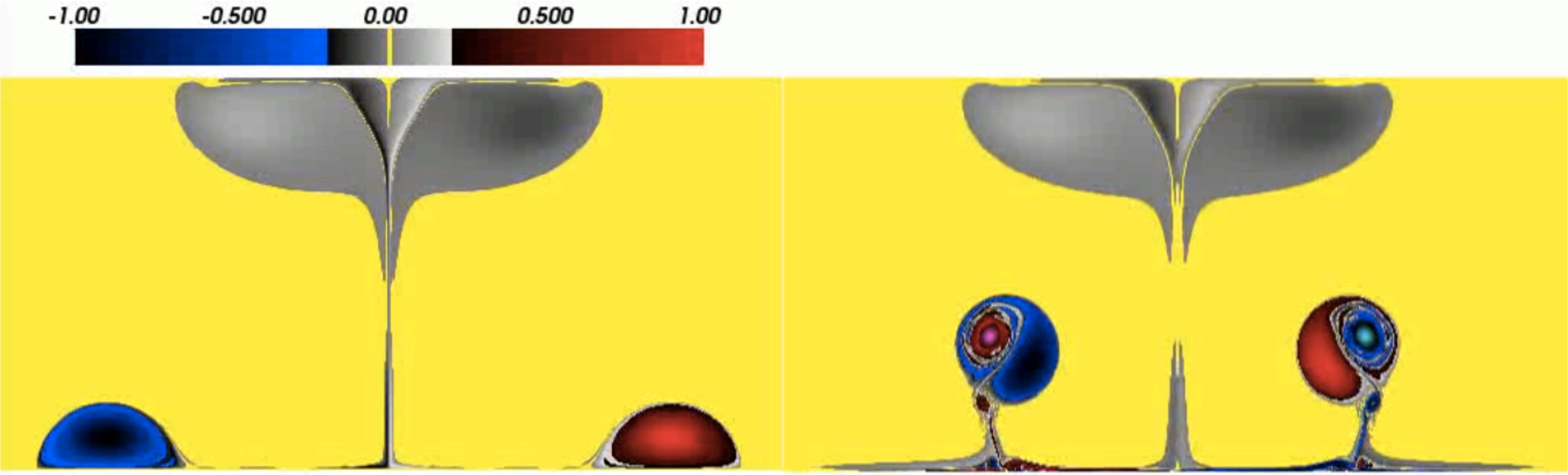




Prandtl’s solution
no more exists
after t= 55.8

Euler

Navier-Stokes



Prandtl's solution
no more exists
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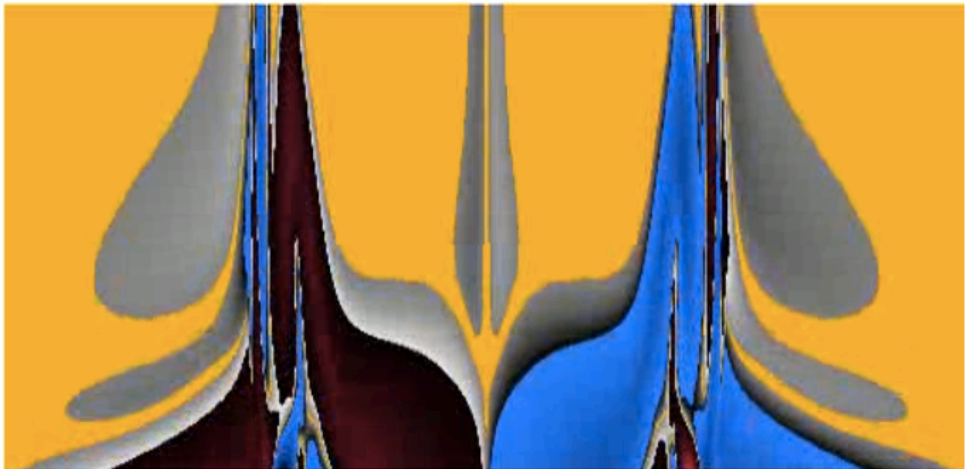
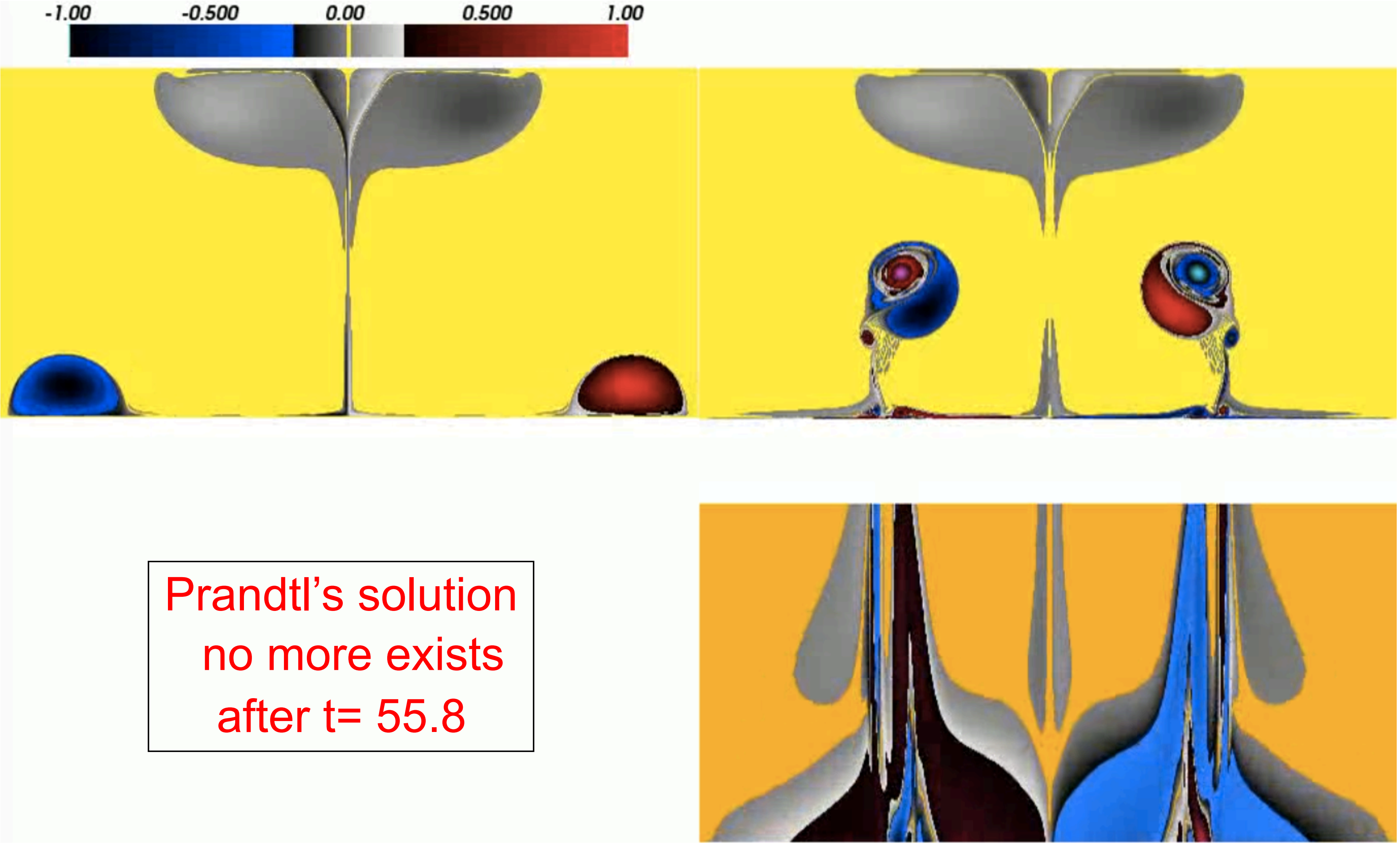


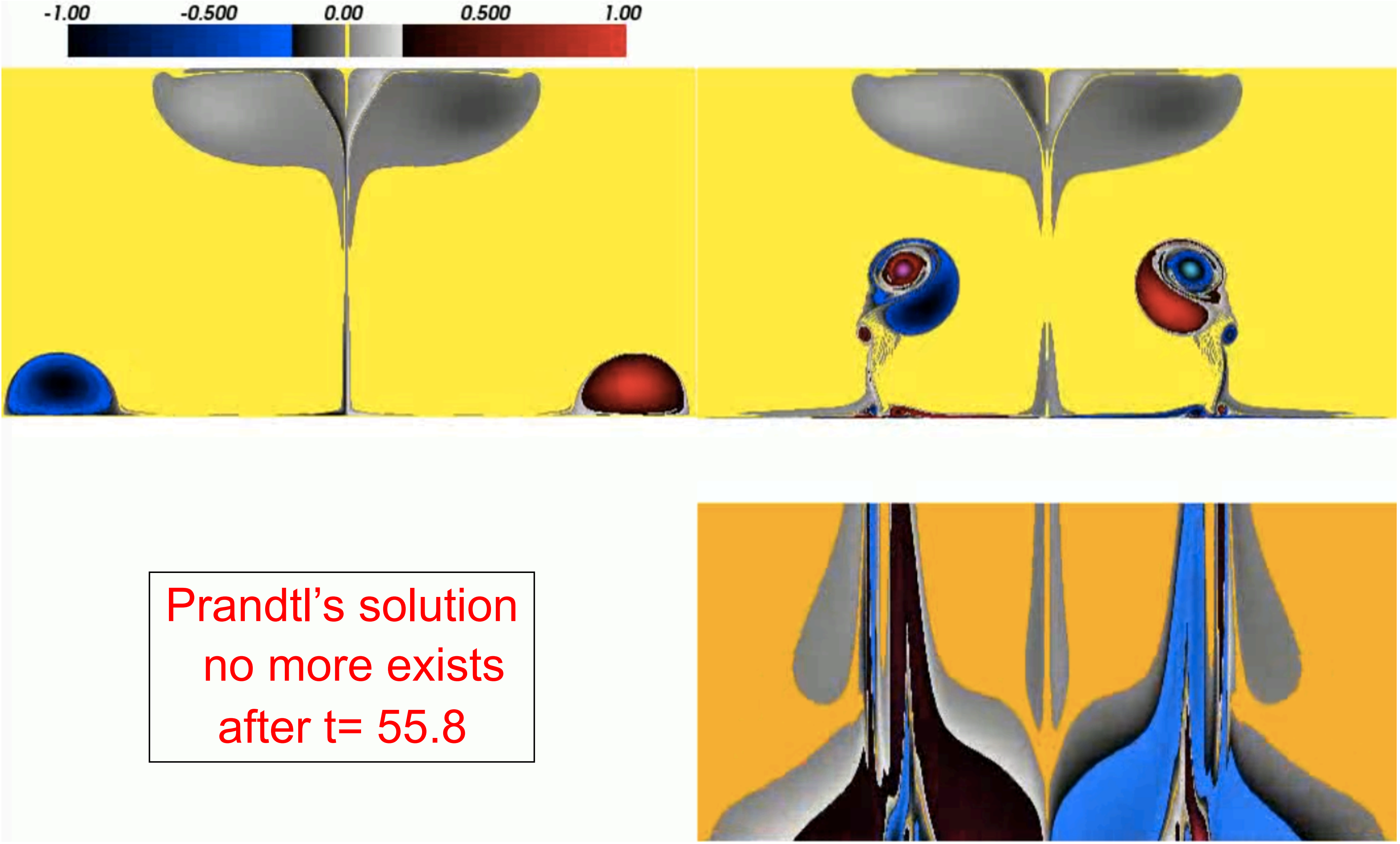
Euler

Navier-Stokes



Prandtl's solution
no more exists
after t= 55.8

Euler **Navier-Stokes**



Prandtl's solution
no more exists
after t= 55.8

Euler

Navier-Stokes

Prandtl's singularity

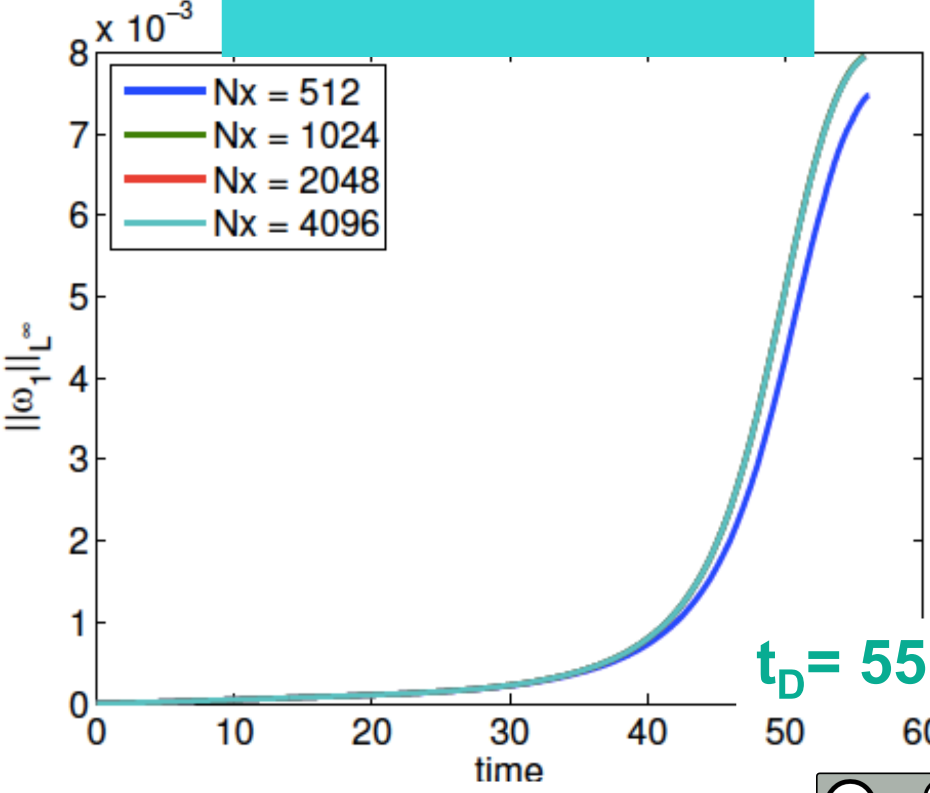
Prandtl equation has well-known finite time singularity

- $|\partial_x \omega_1|$ and $u_{1,y}$ blows up,
- ω_1 remains bounded.

L. L. van Dommelen
and S. F. Shen., 1980
J. Comp. Phys., ***38****(2)*

This is observed
in our computations
as expected,

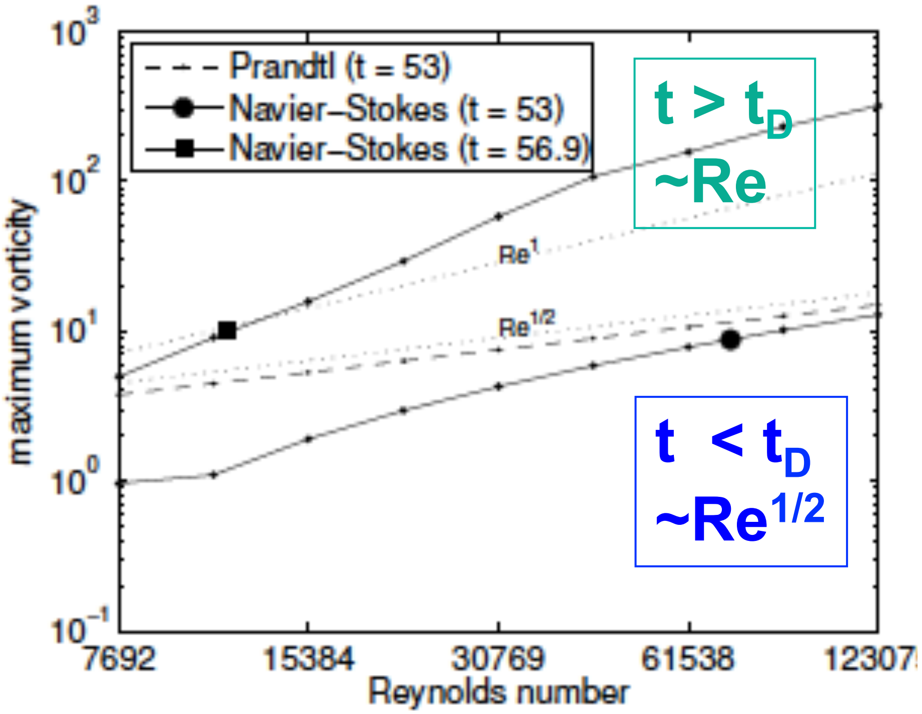
$\boxed{\text{for } t \to t_D \simeq 55.8}$



$t_D = 55.8$

Scaling from Re=7692 to 123075

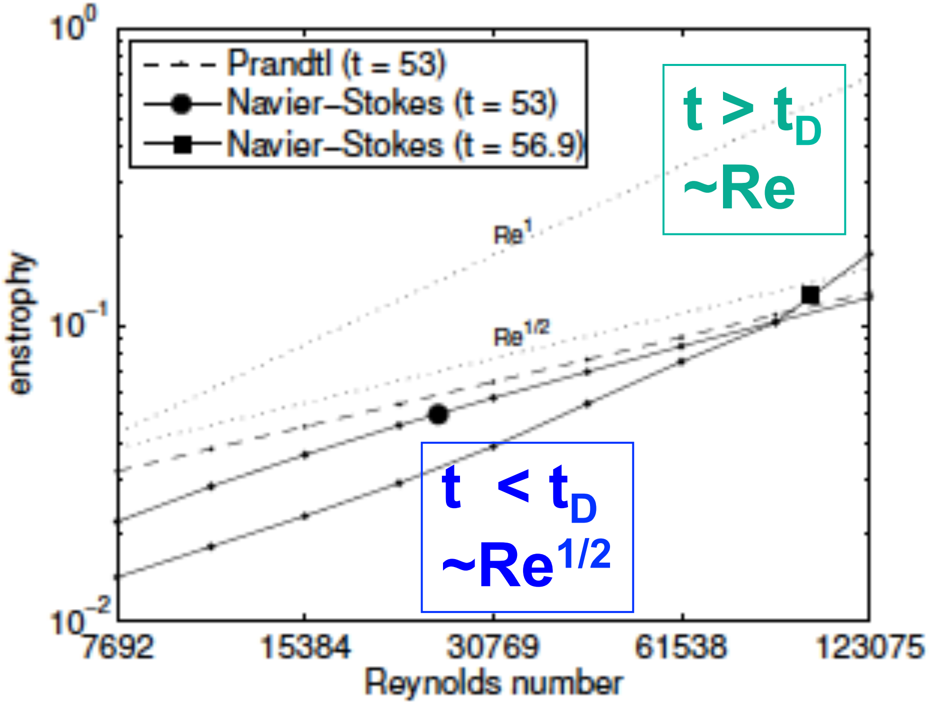
Vorticity max



$t > t_D$
$\sim Re$

$t < t_D$
$\sim Re^{1/2}$

Enstrophy



$t > t_D$
$\sim Re$

$t < t_D$
$\sim Re^{1/2}$

We observe Prandtl’s scaling in $Re^{1/2}$ before $t_D \sim 55.8$ and Kato’s scaling in Re after.

Conclusions

- The Prandtl solution becomes singular at t_D when BL detaches.

- The Navier-Stokes solution converges uniformly to the Euler solution before BL detaches and ceases to converge after BL detaches.

- The BL detachment involves spatial scales as fine as Re^{-1} produced in different directions, not only parallel to the wall, while attached BL is parallel to the wall and scales as $Re^{-1/2}$.

- The maximal vorticity of Navier-Stokes solution does not appear at the same location of the Prandtl singularity. This contradicts the picture of BL detachment seen as a local process coinciding with Prandtl singularity.

Open questions

Numerical results suggest that a new asymptotic description of the flow beyond the breakdown of the Prandtl regime is possible.

Studying it might help to answer the following questions:

- Would Navier-Stokes solution looses smoothness after t_D?
-Would it converges to a weak singular dissipative solution of Euler's equation analog to dissipative shocks in Burgers solution?
- How can such a weak solution be approximated numerically?

This might lead to a new resolution of d'Alembert's paradox in terms of the production of weak singular dissipative structures due to the interaction of fully-developed turbulent flows with walls.

J. Leray, 1934
Sur le mouvement d'un fluide visqueux,
Acta Mathematica, ***63***

C. de Lellis and L. Székzlyhidi, 2010
Archives Rational Mechanics and Analysis,
195*(1), 221-260*

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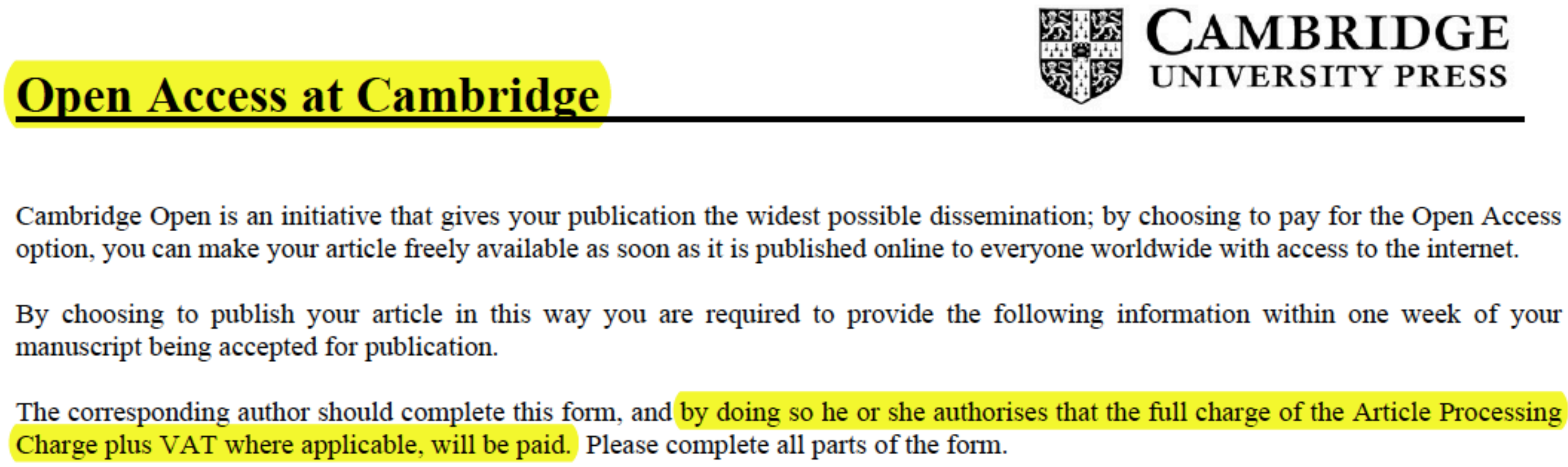

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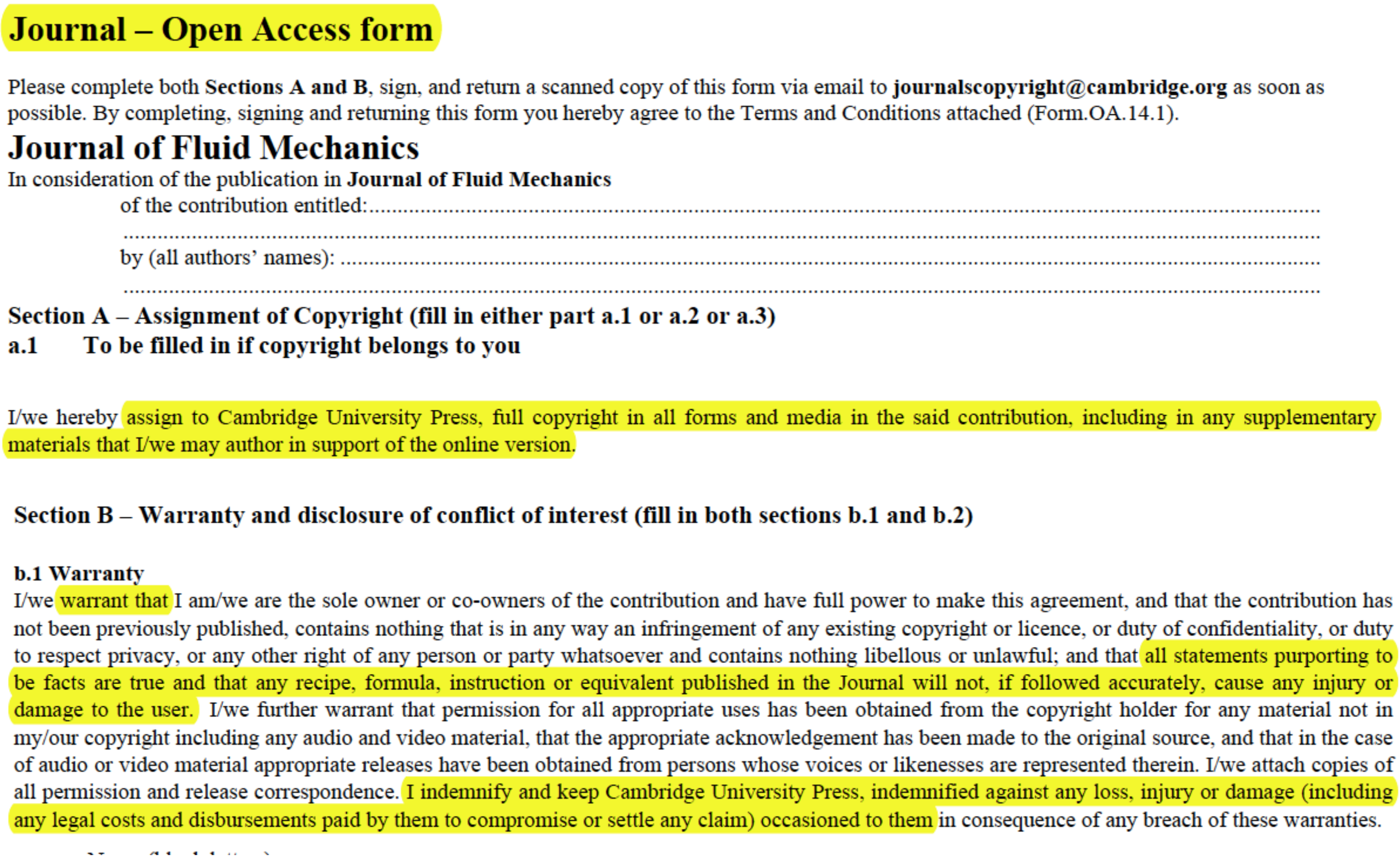

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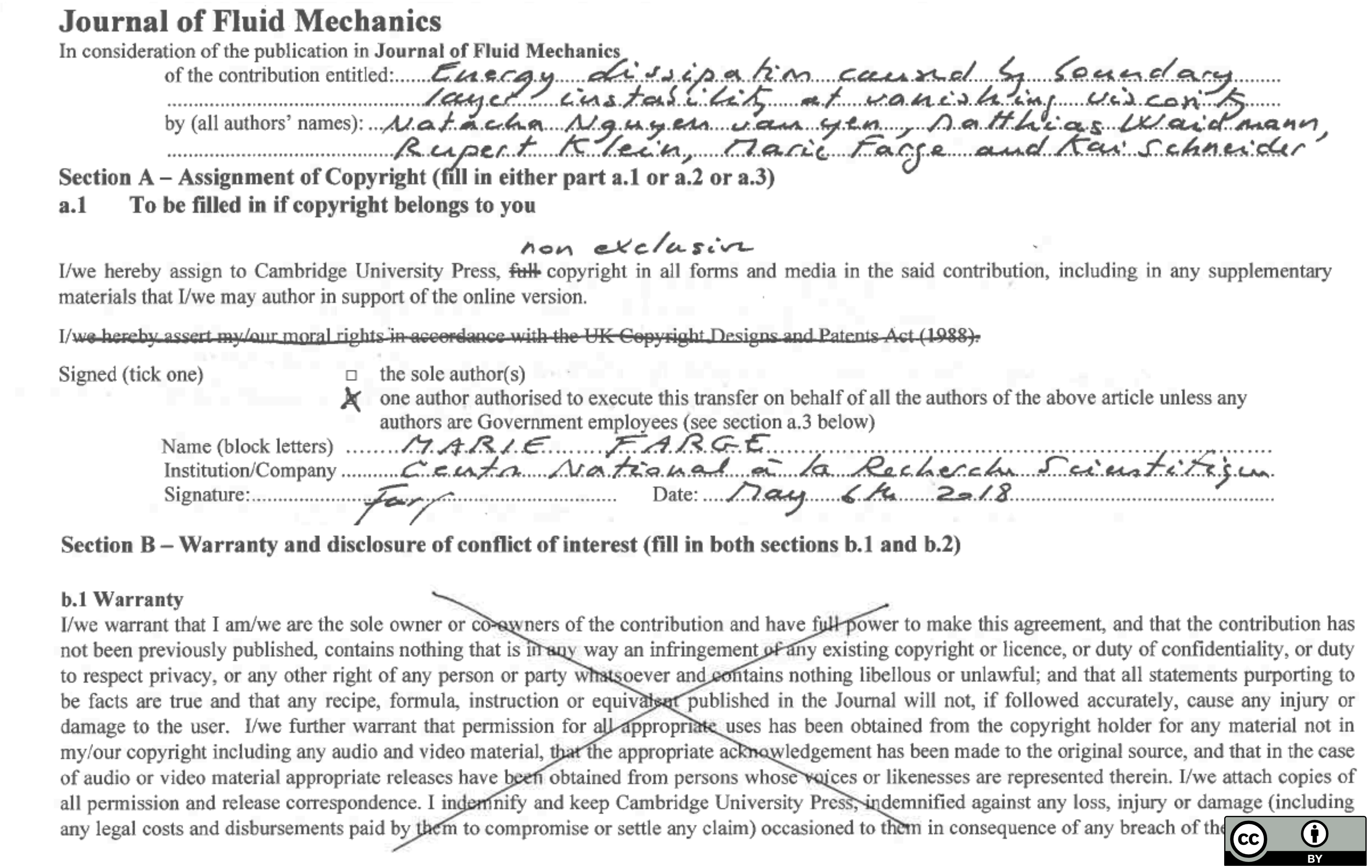

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by (all authors' names): Natacha Nguyen van yen, Matthias Waidmann, Rupert Klein, Marie Farge and Kai Schneider

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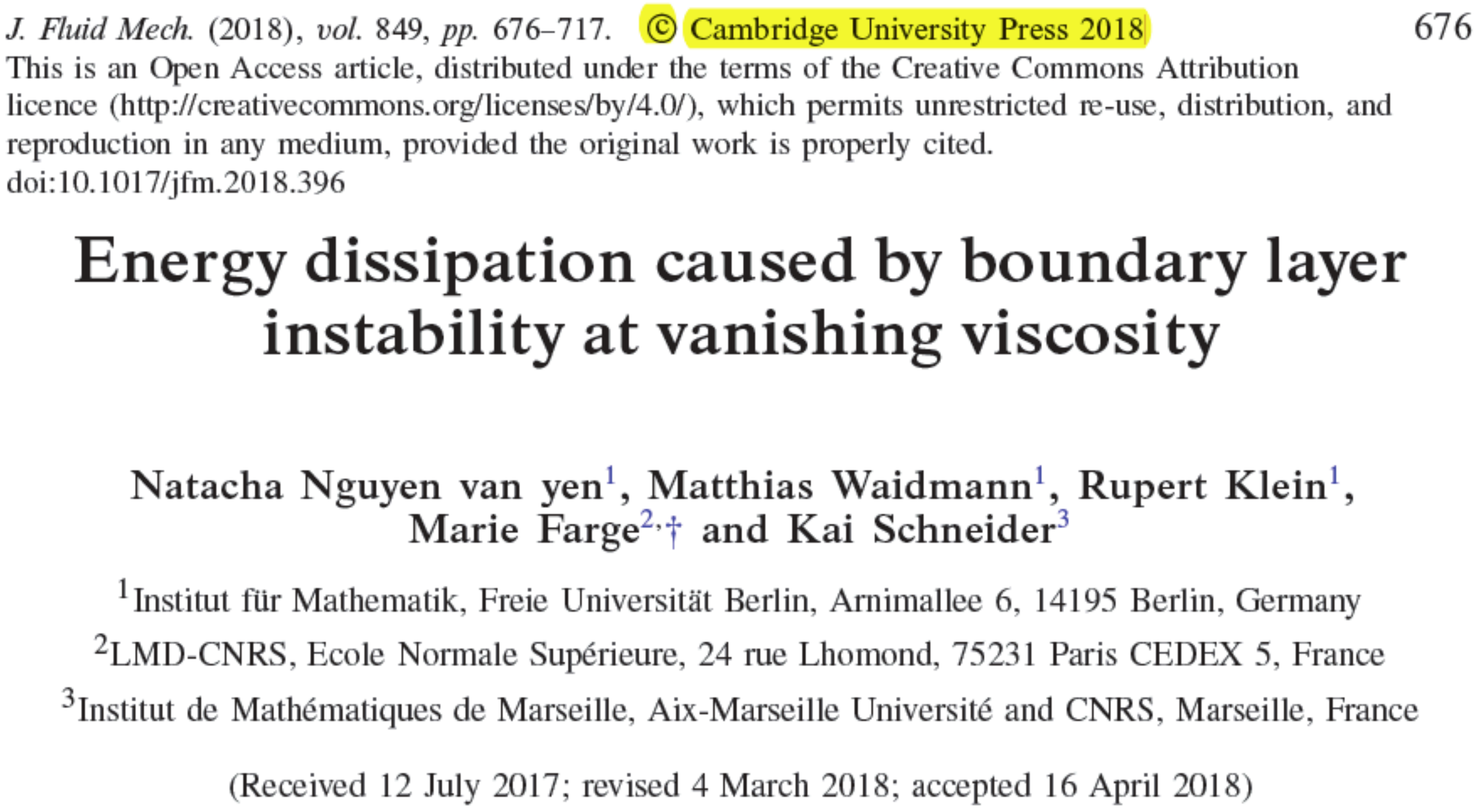

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Energy dissipation caused by boundary layer instability at vanishing viscosity

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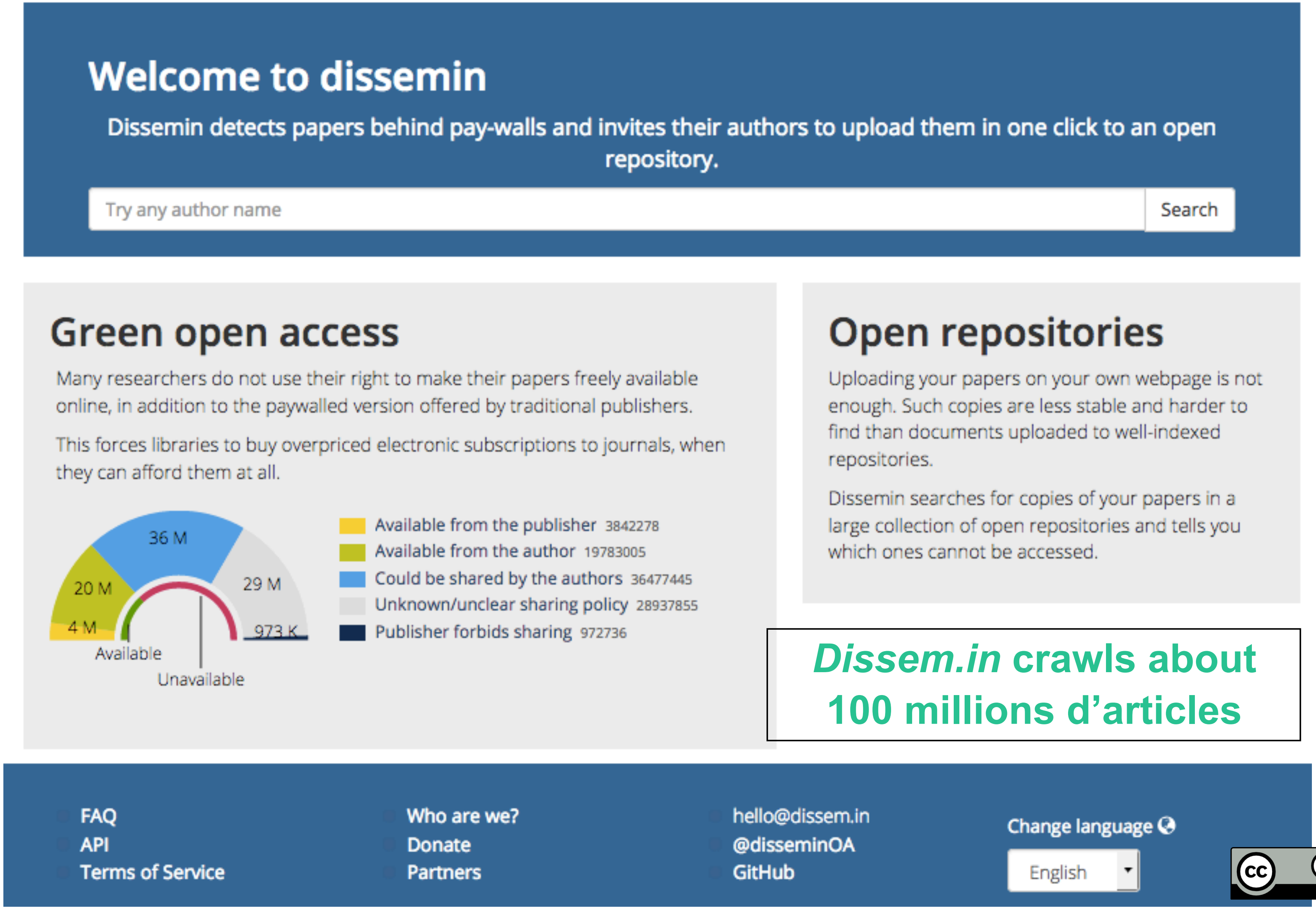

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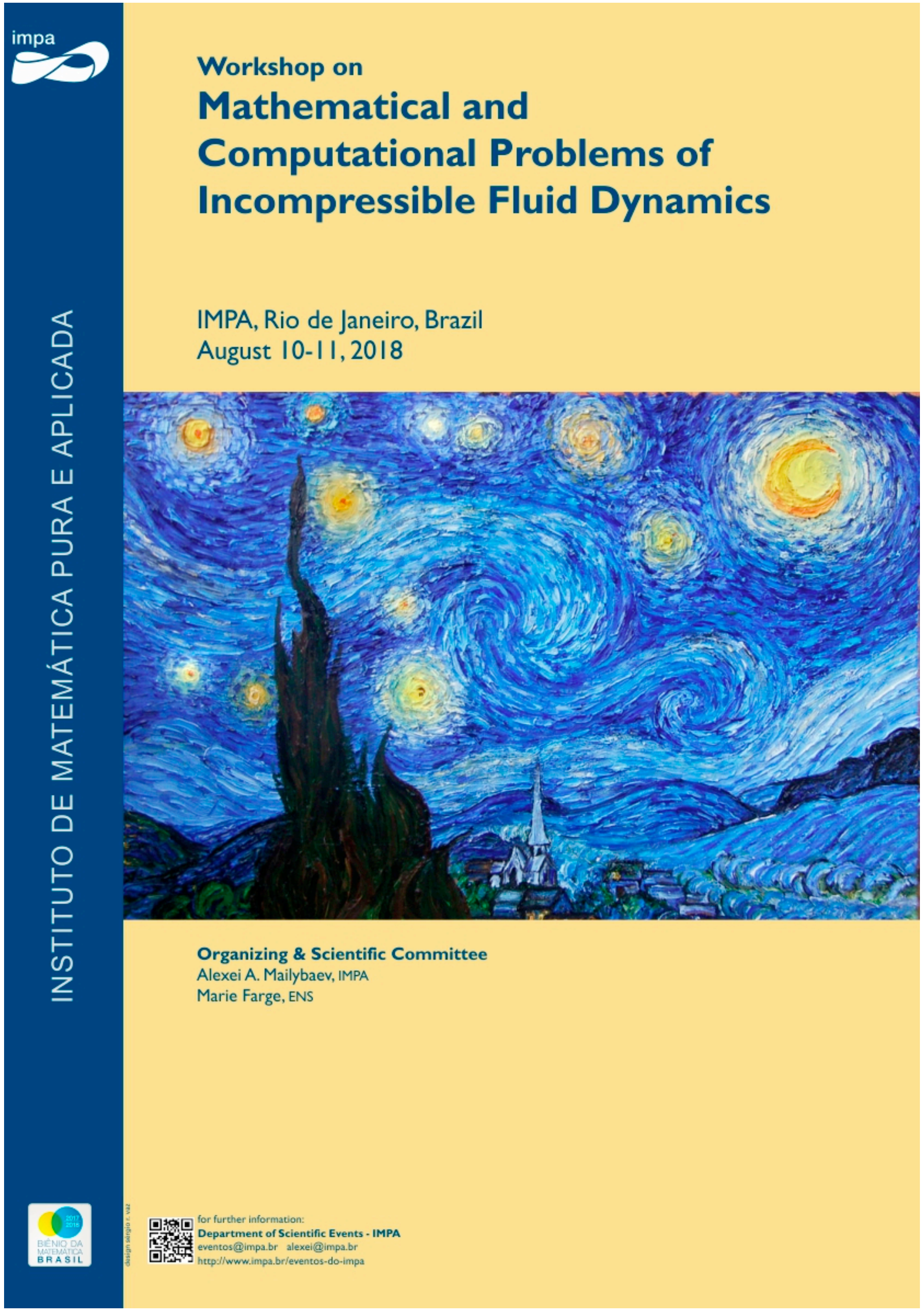

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